Question 1

Find the angle between the vectors \( u = \langle 1, -1, 1 \rangle \) and \( v = \langle -1, 1, -1 \rangle \). (Give the angle in degrees or radians.)

Answer: 

Question 2

Find the area of the parallelogram generated by the vectors \( u = \langle 1, 0, 1 \rangle \) and \( v = \langle 0, 1, 1 \rangle \).

Answer: 
Question 3

Find the parametric equations of the line containing the points \(P(1, -2, 1)\) and \(Q(2, 0, 4)\).

Answer: ..........................

Question 4

Find an equation of the plane that passes through the point \(P(1, -1, 0)\) and is perpendicular (normal) to the line with symmetric equations \(\frac{x - 2}{3} = \frac{y + 5}{2} = \frac{z - 1}{-1}\).

Answer: ..........................

Question 5

Use the Fundamental Theorem of Calculus to find the derivative of the function
\[ g(x) = \int_{0}^{x} \sin(t^3) \, dt. \]

Answer: ..........................
Question 6

Determine whether the improper integral is convergent or divergent. Evaluate the integral if it is convergent.

\[
\int_{1}^{\infty} \frac{1}{\sqrt[3]{x^3}} \, dx
\]

Answer: ..................

Question 7

Find the area of the region bounded by the curves \( y = x^2 \) and \( y = \sqrt{x} \).

Answer: ..................

Question 8

Evaluate the indefinite integral \( \int \sin^3(x) \cos(x) \, dx \).

Answer: ..................
Question 9

Evaluate the indefinite integral \[ \int \frac{x}{x + 2} \, dx. \]

Answer: .................. 

Question 10

Determine whether the alternating series \[ \sum_{n=1}^{\infty} (-1)^n \frac{n - 1}{n^2 + 2} \] is divergent, absolutely convergent, or conditionally convergent. (Be specific!)

Answer: ..................
Problem 1

Two planes are given by the equations $x + y + z = 1$ for the plane $P_1$ and $x - y + z = 1$ for the plane $P_2$.

(a) Find the coordinates of a point of intersection of the planes $P_1$ and $P_2$.

(b) Find the normal vector (i.e., the vector perpendicular) to the plane $P_1$ and the normal vector to the plane $P_2$.

(c) Find the parametric equations of the line of intersection of the planes $P_1$ and $P_2$. 
Problem 2

This problem has two separate questions. (Answer all the questions!)

(a) Find the length of the arc of the circular helix with vector equation
\[ \mathbf{r}(t) = (\cos(t), \sin(t), 3t) \] when \( 0 \leq t \leq 3 \).

(b) Determine whether the (improper) integral
\[ \int_{0}^{\infty} 2x e^{-x^2} \, dx \]
is convergent or divergent. Evaluate the integral if it is convergent.
Problem 3

Evaluate the following integrals (clearly show the techniques of integration you use):

(a) \[ \int \frac{1}{x \ln^3(x)} \, dx \]

(b) \[ \int x e^x \, dx \]

(c) \[ \int \frac{3x^2 + 5x + 3}{(x + 2)(x^2 + 1)} \, dx. \]
Problem 4

This problem has two separate questions. (Answer all the questions!)

(a) Find the area of the region enclosed by the parabola $y = x^2 - x$ and the parabola $y = x - x^2$.

(b) The region enclosed by the curves $y = \sqrt{x}$ and $y = x^2$ is rotated about the $x$-axis. Find the volume of the solid obtained in this way.
Problem 5

This problem has two separate questions. (Answer all the questions!)

(a) Find the radius and interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3} (x - 3)^n.$$ 

Be sure to check any endpoints that exist!
(b) Use the Maclaurin series of the function \( \sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n + 1)!} \) to first write out the Maclaurin series for the function

\[ g(x) = \sin(x^2), \]

and then write out the Maclaurin series expansion of

\[ \int_0^1 \sin(x^2) \, dx. \]

(Do not compute and add up the terms of your series!)

Using the above information, find the minimum number of terms needed to approximate \( \int_0^1 \sin(x^2) \, dx \) with an error less than \( 0.0001 = \frac{1}{10,000} \).
Summary of scores on problems - for grading purposes only.

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SCRATCH PAPER

(Scratch paper will not be graded)
CALCULUS II, FINAL EXAM

SCRATCH PAPER

(Scratch paper will not be graded)
SCRATCH PAPER

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