Show all your work and justify your answer!
No partial credit will be given for the answer only!

PART I

You must simplify your answer when possible.
All problems in Part I are 10 points each.

1. Find the derivative of the function \( y = f(x) = \cos(x^3) \).

2. Find the derivative of \( f(x) = (x^2 + x)^8 \).
3. Find the absolute maximum and minimum of the function 
   \[ y = f(x) = (2x - 3)^2(x + 1)^5 \] on the interval \([0, 1]\).

4. Find the linearization of the function \( f(x) = x \tan(x) \) at the point \( a = \pi/4 \) and use it to estimate the value \( f(.8) \).
5. Find two positive numbers so that their sum is 200 and their product is maximal. [As always you must justify your answer!]

6. Suppose that the \textbf{derivative} of a function $y = f(x)$ is given:

\[ f'(x) = (x + 2)(3 - x). \]

(a) Find the $x$-coordinates of all local max/min of the function $y = f(x)$.

(b) At which $x$ value is the function $y = f(x)$ most rapidly increasing?
7. **[15 points]** You work for a soup company. In order to maximize visibility of the product on the shelf your boss asks you to design a soup can of volume $1 \text{dm}^3$ and maximal surface area. Either specify the dimensions of such a can or show that such a can does not exist.

You may use that the volume of a can of radius $r$ and height $h$ is $V = \pi r^2 h$ while the surface area of the side is $2\pi rh$ and of the top (and bottom) is $\pi r^2$. 
8. [20 points] Use calculus to graph the function \( y = f(x) = \frac{x}{x^2 + 1} \). Indicate

- \( x \) and \( y \) intercepts,
- vertical and horizontal asymptotes (if any),
- in/decreasing; local/absolute max/min (if any).

You must show work to justify your graph and conclusions. You can use decimal numbers to plot points (but mark them with exact values).
9. [5 points] Find the equation of the tangent line to the graph of \( x^2 + y^3 = 2xy \) at the point \((1, 1)\).