TEST IV

Show all your work! No partial credit will be given for the answer only!

PART I

Part I consists of 8 questions. Clearly write your answer in the space provided after each question. Show all of your work!

All problems in Part I are 8 points each.

Question 1

Given the graph of the function below find

1. \( f(2) \)
2. Is \( f \) one-to-one?
3. \( f^{-1}(1) \)
4. \( (f^{-1})'(1) \)
Question 2

Find the derivative of $f(x) = xe^x$

Question 3

Find the derivative of $y = f(x) = \ln(x^4)$.

Question 4

Evaluate $\int e^{-x} \, dx$

Question 5

Evaluate $\int e^{\sin(x)} \cos(x) \, dx$
Question 6
Solve $\ln(2x + 5) = 3$

Question 7
Use Newton’s method to find the second approximate solution $x_2$ of the equation $f(x) = x^3 + \sin(x) - \frac{1}{10} = 0$ if $x_1 = 0$

Question 8
Show that the function $y = f(x) = x^3 + 1$ is one-to-one and find its inverse function $f^{-1}(x)$. 
PART II

Part II consists of 3 problems. You must show correct reasons to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit.

Problem 1 (12 points)

Evaluate

\[ \int_{1}^{2} \frac{\sin(x) \cos(x)}{\sin^2(x) + 1} \, dx \]
Problem 2 (12 points)

Use logarithmic differentiation to find the derivative of

\[ f(x) = \frac{\sqrt{x^2 + 1}}{2x - 5} \]

You must simplify your answer!
Problem 3 (12 points)

Find the absolute maximum and minimum of the function $y = f(x) = x^2 \ln(x)$ on the interval $\left[\frac{1}{10}, 2\right]$. 