Part I consists of 5 questions. Clearly write your answer (only) in the space provided after each question. Show your work to justify your answers. Very limited partial credit or none at all for this part of the test!

Each question is worth 8 points.

Question 1

Determine whether the improper integral \( \int_{2}^{\infty} \frac{2x}{(x^2 - 3)^{\frac{3}{2}}} \, dx \) is convergent or divergent. Find its numerical value if it converges!
Question 2

Sketch the region enclosed by the curves $y = |x|$ and $y = 6 - x^2$, and then find its area.

Answer: ..................

Question 3

Find the volume of the solid obtained by rotating about the $y$-axis the region bounded by the curve $y = \sqrt{x}$ and the lines $x = 0$ and $y = 1$.

Answer: ................
Question 4

Sketch the region described herein, and use the method of cylindrical shells to write out an integral-formula for the volume of the solid generated by rotating about the $y$-axis the region bounded by the curve $y = e^x$ and the lines $y = x$, $x = 0$, and $x = 1$. (Do NOT compute the integral you obtain!)

Answer: .................

Question 5

When a particle is located a distance $x$ meters from the origin, a force $f(x) = \cos(\pi x/3)$ newtons acts on it. How much work is done in moving the particle from $x = 1$ m to $x = 2$ m? (Express your answer in Nm; i.e., in J.)

Answer: .................
Part II consists of 4 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit - no credit for unsubstantiated answers!

Problem 1

The graph of the acceleration $a(t)$ of a car measured in $\text{ft/s}^2$ is shown. Use this graph to set up and write out the mid-point rule approximation $M_6$ in order to estimate the increase in the velocity of the car during the 6-second time interval. (Do NOT add up the terms!)
Problem 2

Determine how large the number $a$ has to be so that the area under the graph of the function $f(x) = \frac{1}{1 + x^2}$ is less than or equal to $\pi/3$ for $x \geq a$; that is, determine how large the number $a$ has to be so that

$$\int_{a}^{\infty} \frac{1}{1 + x^2} \, dx \leq \frac{\pi}{3}.$$ 

(Give the exact answer! NO approximation.)
Problem 3

Find the volume of the solid obtained by rotating the region bounded by the curves $y = x^2$ and $y = \sqrt{x}$ about the line $y = -1$. 
Problem 4

A force of 14 lb is required to hold a spring stretched from its natural length of 5 in. to a length of 9 in.

(a) Find the force $f(x)$ required to hold the spring stretched $x$ ft beyond its natural length. (1 ft = 12 in.)

(b) How much work $W$ is done in stretching the spring from 9 in. to 14 in.? (Express your answer in ft-lb units.)
SCRATCH PAPER

(Scratch paper will not be graded!)
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(Scratch paper will not be graded!)