CALCULUS I
Final Exam, Wednesday, December 9, 2015

Name (Print last name first): ...........................................
Instructor: .................................................................

Show all your work and justify your answer!
No partial credit will be given for the answer only!

PART I

You must simplify your answer when possible but don’t simplify numbers!
All problems in Part I are 6 points each.

1. Use the definition of the derivative to show that \((2x + 1)' = 2\).

2. Find the derivative of \(f(x) = x^3 \sin(x)\).
3. Find the derivative of \( f(x) = \sec(x^3) \).

4. Find the derivative of \( f(x) = \frac{x^2 + 1}{x^2 - 1} \).

5. Find the derivative of \( f(x) = \int_{5}^{x} \sin(t^3 + t) \, dt \).
6. Evaluate $\int x(x^2 + 1) \, dx$.

7. Evaluate $\int \frac{x^2+1}{x} \, dx$.

8. Evaluate $\int x^3 \sin(x^4 + 5) \, dx$. 
9. Use a Riemann sum with 4 terms and the midpoint rule to approximate the value of \( \int_{1}^{3} \sin(x^3 + x) \, dx \). [You do not need to add the resulting sum of numbers; i.e., \( e^{-7} + \ln\left(\left(\frac{5}{3}\right)^4 + 9\right) \) could be a complete answer!]

10. Use Newton’s method to compute the second approximate solution to the equation

\[
f(x) = \cos(x) - x = 0
\]

if the first approximate solution \( x_1 = 0.6 \).
[PART II]

All problems in Part II are 10 points each.

1. Evaluate \( \int_1^e \frac{[\ln(x)]^2}{x} \, dx \)

2. Find the absolute maximum and minimum of the function

\[ f(x) = (1 - 2x)^3(1 + x)^5 \]

on the interval \([0, 1] \).
3. Find the dimensions of a box, with a square base, of volume $V = 1 \, m^3$ of minimal cost if the material for the top costs $\$2$ per square meter and the material for the sides and bottom costs $\$10$ per square meter.

\[ V = x^2 \cdot y = 1 \]
4. Graph the function $f(x) = \frac{x^2 - 1}{x+2}$. Indicate $x$ and $y$ intercepts, horizontal and vertical asymptotes (if any). Find intervals where the function is in-/decreasing and mark any Local/Absolute Max/Min on the graph.

You can use decimal numbers and your calculator to compute approximate values—additional scratch paper is on the back of this page.

Use the coordinate system on the next page to draw your graph.