The Final Grade for TEST 1:

I. (10%) A function \( y = f(x) \) is graphed below.

a) Estimate as well as you can:
\[
\lim_{x \to -2^-} f(x) = \\
\lim_{x \to -2^+} f(x) = \\
\lim_{x \to -2} f(x) = \\
\lim_{x \to 0^-} f(x) = \\
\lim_{x \to 0^+} f(x) = \\
\lim_{x \to 5^-} f(x) = \\
\lim_{x \to 5^+} f(x) = \\
\lim_{x \to 7} f(x) = 
\]

b) At which points is the function graphed is discontinuous and why?
II. (10%) a) Find the limit:
\[ \lim_{x \to 3} \frac{x^2 - 9}{x - 3}. \]
b) Explain, why the function
\[ f(x) = \begin{cases} 
\frac{x^2 - 9}{x - 3}, & x \neq 3, \\
0, & x = 3,
\end{cases} \]
is discontinuous at \( x = 3 \). Sketch the graph of the function.
III. (10%) Prove that the equation

\[
\frac{x^5 + 6x - 1}{3x^4 + 14x^2 + 51} = \frac{1}{100}
\]

has at least one solution in the interval [0,1].
IV. (10%) Let 
\[ f(x) = \frac{x^6 + 3x - 7}{4x^2 + 1 - 3x^6}. \]

a) Find the limit: \( \lim_{x \to \infty} f(x) \).
b) Find the horizontal asymptotes of \( f(x) \) both at \( +\infty \) and \( -\infty \).

V. (10%) Find the limit:
\[ \lim_{x \to -\infty} \frac{x^5 + 6x - 1}{3x^4 + 14x^2 + 51}. \]
VI. (10%) Find the limit:

\[
\lim_{x \to \infty} \frac{x^2 + 500}{x^3 - 99}.
\]

VII. (10%) Let

\[ f(x) = \frac{x}{x - 3}. \]

Find:

a) \( \lim_{x \to 3^-} f(x) \).

b) \( \lim_{x \to 3^+} f(x) \).

c) A vertical asymptote of the function.
VIII. (10%) The curve below represents the graph of a function $f(x)$. Use the graph to estimate $f'(x)$ at the point $x = 23$. Explain your answer.
IX (10 %). Sketch the graph of a function $f(x)$, which satisfies the conditions: $f'(-2) = -1$, $f'(0) = 0$, $f'(2) = 1$.

X (10 %). The limit below represents the derivative of some function $f(x)$ at some number $a$. State $f$ and $a$:

$$
\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4}.
$$