• Show your work; clearly write down each step in your calculation/reasoning. *No credit* is given for a correct numerical answer without any justification.
1. (a) (4pts) Differentiate
   \[ f(x) = (x^2 - 45)^3 \]

(b) (6pts) Find the points of inflection of \( f \).

2. (10pts) Differentiate
   \[ f(x) = \frac{e^{3x}}{1 + \ln x} \]

3. (10pts) Differentiate
   \[ y = \sin^2 (3x^2 - 2x) \]
4. (10pts) Differentiate \( y = (\cos x)^x \)

(Hint: use logarithmic differentiation)

5. (10pts) Use logarithmic differentiation to differentiate the following:

\[
y = \frac{e^x(2x + 1)\sqrt{x}}{x^2 - 1}
\]

(You don’t have to find a common denominator.)
6. (5pts) Suppose that $h(x) = f(g(x))$. Find $h'(1)$ if $f'(5) = 2$ and $g(x) = 4x^2 + x$.

7. (10pts) Find the point(s) of the curve $y = e^x(x^2 + 4x + 5)$ where the tangent line is horizontal.
8. (10pts) Find the equation of the tangent line at the point \((-5, \frac{9}{4})\) for the following curve:

\[
\frac{x^2}{16} - \frac{y^2}{9} = 1
\]

9. (a) (7pts) Use Newton’s method with \(x_1 = 1.2\) to show that \(\sqrt{2} \approx 1.18920712\).

(b) (3pts) Starting with \(x_1 = 1.2\), at what \(x_n\) can you stop, if you would like to know \(\sqrt{2}\) correct to 6 decimal places? Explain your answer.
10. The revenue $R(x)$ for a company when producing $x$ units is given by the equation

$$R(x) = xp(x)$$

where $p(x)$ is the selling price when the company produces $x$ units. The function $p(x)$ is assumed to be differentiable.

(a) (5pts) Suppose that $p(1000) = 10.50$, $p'(1000) = 0.002$. What is $R'(1000)$?

(b) (5pts) Use linear approximation to estimate $R(1005)$, the revenue when producing 1005 units.

(c) (5pts) Suppose the company is producing $x$ units. Use differentials to approximate the change in revenue ($\Delta R$) when the company will start to produce $x + 1$ items. Express your answer in terms of $p(x)$ and $p'(x)$ only.