Calculus I

FINAL EXAM
Version A

December 13\textsuperscript{th}, 2004

Name: __________________________

- Show your work; clearly write down each step in your calculation/reasoning. \textit{No credit} is given for a correct numerical answer without any justification.
1. (12 pts) Evaluate the following limits:
   (a) \( \lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4} \)
   (b) \( \lim_{x \to \pi} \frac{3}{\sin^2 x} \)
   (c) \( \lim_{x \to 0^+} (1 + 3x)^{\frac{1}{x}} \) \{Hint: use the fact that \( f(x) = e^{\ln f(x)} \}\)

2. (10 pts) Find the equation of the tangent line to the graph of \( y = f(x) = \frac{x}{x + 2} \) at the point (-1,-1).
3. (8pts) Differentiate 
\[ x^2 \ln x + e^{x^2} \]

4. (8pts) Differentiate 
\[ \cos(x + 2x^2) \]

5. (10pts) Find \( y' \) using implicit differentiation if \( 2xy + x^2 = e^y \).
6. (14pts) Use calculus to determine intervals of increase or decrease, local maxima/minima, intervals of concavity and inflection points of the function $f(x) = \frac{1}{5}x^5 + \frac{1}{2}x^4 - x^3 + 2$. (You may round off the y-coordinate of the inflection point to 2 decimals). Use this information to sketch the graph of $f(x)$. 
7. (a) (7pts) Let \( f(x) = \sqrt[3]{4x-4} \). Find the linear approximation of \( f(x) \) at \( x = 3 \).

(b) (3pts) Use the above linear approximation to estimate \( f(2.97) \).

8. (10pts) Find the point on the line \( y = 2x + 2 \) that is closest to the point \( (5,2) \).
9. (8pts) Find the most general antiderivative of:
\[
\frac{2}{\sqrt{1-x^2}} + \cos x + e^x
\]

10. (10pts) Evaluate the following integral:
\[
\int_1^3 \frac{1}{3x} + 3x^2 \, dx
\]

(BONUS, 2pts) Calculate the derivative of \( f(x) = x^2 \).

(BONUS, 3pts) Calculate \( \int_{-2}^2 f(x) \, dx \) if \( f(x) \) is given by
\[
f(x) = \begin{cases} 
\sqrt{4-x^2} & \text{if } -2 \leq x \leq 0 \\
2x + 2 & \text{if } x \geq 0
\end{cases}
\]