• Calculators are allowed only for numerical calculations, that is you may not graph functions on your calculator.

• There are two sheets of scratch paper attached at the end of the exam. Use them and but do not tear them off the exam.

• Show your work; clearly write down each step in your calculations/reasonings. No credit is given to a correct numerical answer without any justification.
1. (25 pts) Find the following limits if they exist. The values of the limits may take $+\infty$ or $-\infty$. Justify your answers for each case.

a) \[
\lim_{x \to 1} \frac{1}{x^2 - 4}.
\]

b) \[
\lim_{x \to 2^+} \frac{1}{x^2 - 4}.
\]
c) 
\[ \lim_{{x \to 2}} \frac{{x - 2}}{{x^2 - 4}}. \]

d) 
\[ \lim_{{x \to \infty}} \sqrt{x + 1} - \sqrt{x}. \]
\[ \lim_{x \to +\infty} \frac{x^2 + 4}{x^2 - 4} \]
2. (20 pts) a) Show that the function $|x - 6|$ is not differentiable at $x = 6$.

b) Find a formula for $f'(x)$ where it is defined, and sketch its graph.
3. (20 pts) a) \( f \) is defined as follows;
\[
f(x) = \begin{cases} 
  x \sin \frac{1}{x} & \text{when } x \neq 0 \\
  0 & \text{when } x = 0
\end{cases}
\]
Show \( f \) is continuous at \( x = 0 \). (Hint: use the Squeeze Theorem)

b) \( g \) is defined as follows;
\[
g(x) = \begin{cases} 
  x^2 \sin \frac{1}{x} & \text{when } x \neq 0 \\
  0 & \text{when } x = 0
\end{cases}
\]
Show \( g \) is differentiable at \( x = 0 \). (Hint: use a))
4. (15 pts) Show that there exists a number $x$ whose cube is exactly one more than its square.
5. (20 pts) **a)** Use the definition of a derivative to find $f'(4)$, where $f(x) = \sqrt{x}$.

**b)** Find an equation of the tangent line to the curve $y = \sqrt{x}$ at the point $(4, 2)$.
6. (20 pts) Water is flowing into three containers of different shapes; A) pinched neck, B) cylindrical and C) conical (point-down). Let $H_A(t)$ represent the height of the water level of the container A at time $t$, and $H_B(t)$ $H_C(t)$ defined accordingly.

**a)** For each function, select a graph which best represents its behavior from the six graphs below.

**b)** For the derivative of each function (that is, $H'_A(t)$, $H'_B(t)$ and $H'_C(t)$) select a graph which best represent its behavior from the six graphs below.

Remark: You may want to compare the answers from a) and b) to see they are indeed consistent.