Mathematics 125 **FINAL EXAMINATION**
April 29, 2004

- Calculators are allowed *only* for numerical calculations, that is you may not graph functions on your calculator.

- There are a sheet of scratch paper attached at the end of the exam. Use it but do not tear them off the exam.

- Show your work; clearly write down each step in your calculations/reasonings. *No credit* is given to a correct numerical answer *without* any justification.
1. (20 pts) Differentiate the following functions.
   a) \[
   \frac{1 + x^2}{1 + x}
   \]
   b) \[\cos(\tan x)\]
c) \[ \frac{1}{(1 + x^3)^{10}} \]

d) \[ 10^{x^2} \]

Hint: 10 = \( e^{\ln 10} \).
2. (10 pts) Find values of the following limits

a)  \[ \lim_{x \to 2} \frac{x^2 - 4}{x^2 - x - 2} \]

b)  \[ \lim_{x \to 0^-} \frac{|x|}{x} \]
3. (10 pts) Evaluate the following definite integrals.

a) \[ \int_{1}^{4} \frac{1}{\sqrt{x}} \, dx \]

b) \[ \int_{0}^{2\pi} \cos x \, dx \]
4. (10 pts) A function $f$ is defined as follows,

$$f(x) = \begin{cases} \frac{x^3 - 2x^2}{x^2 - 2} & \text{if } x \neq 2 \\ A & \text{if } x = 2 \end{cases}$$

for some number $A$. a) Write down the definition that $f$ is continuous at $x = 2$.

b) Choose a suitable value of $A$ so that $f$ becomes continuous at $x = 2$. 
5. (10 pts) The table shows the population of Nepal (in millions) as of June 30 of the given year. Use a linear approximation to estimate the population at midyear in 1989.

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<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>P(t)</td>
<td>15.0</td>
<td>17.0</td>
<td>19.3</td>
<td>22.0</td>
</tr>
</tbody>
</table>
6. (20 pts) Consider $f(x) = x^4 - 6x^2$.

a) Find the intervals on which $f$ is increasing and the intervals where $f$ is decreasing.

b) Find the intervals where the graph of the function is concave up and the intervals where the graph of $f$ is concave down.
c) Using the information obtained above, sketch a graph of $f$.

d) Find the local maximum and minimum values of $f$. Are they the absolute maximum or minimum?
7. (15 pts) A box with a square base and open top must have a volume of $32\text{cm}^3$. Find the dimension of the box (i.e. the height and the side length of the square) that minimizes the amount/area of the material to make the box.
8. (15 pts) A manufacturer produces a fabric with a fixed width. The quantity \( q \) of this fabric (measured in yards) that is sold is a function of the selling price \( p \) (in dollars per yard), which we write as \( q = f(p) \). (For example \( f(15) = 15,000 \) means if the price is set $15 per yard, then the manufacturer sells 15,000 yards of the fabric.) a) Suppose we know \( f(20) = 10,000 \) and \( f'(20) = -350 \). Find an approximate value of \( f(21) \).

b) Note that the total revenue earned with selling price \( p \) is \( R(p) = p \cdot f(p) \). Find the value of \( R'(20) \) and interpret your answer. In particular, as far as increasing the revenue, is raising the price from 20 a good move?
9. (10 pts) The velocity function $v(t)$ of a car making a round-trip from Birmingham to Atlanta is given as below.

a) Estimate the distance between Birmingham and Atlanta using the graph of the velocity function.
b) Anniston is 50 miles away from Birmingham. When does the car pass by Anniston on the way back?