MA 125-5B, Spring 2004

TEST # 3
March 31, 2004 (70 minutes)

Name: SSN:

Max. Points: 100 + 5 Bonus Points: Test Grade:

Turn in all the work which you did to solve the problems, not just the final answer. In particular, include intermediate steps in calculations, wherever they are needed. You may write on the back of a page if you need extra space.

No book, no notes, and no calculator are to be used!

1. Find \( f'(x) \) for the following functions (3 \( \times \) 5P):
   
   (a) \( f(x) = xe^{3x} \)

   (b) \( f(x) = \ln(x + \sqrt{x}) \)
(c) \( f(x) = \left( \frac{\sin x}{\cos x - 1} \right)^2 \)

2. Evaluate the following limits (3 \( \times \) 5P):

(a) \( \lim_{x \to 0} \frac{e^x - 1 - x}{x^2} \)

(b) \( \lim_{x \to \infty} e^{-x} \ln x \)

(c) \( \lim_{x \to 1} \frac{x^2}{x + 1} \)
3. (a) Find the linearization of \( f(x) = \sqrt{4 + x} \) at \( a = 0 \). (8P)

(b) Use the result from (a) to find an approximate value for \( \sqrt{4.2} \). (4P)

4. (a) In the graphs below use Newton’s Method to graphically determine an approximation \( x_2 \) for the root of \( f(x) \) from the given rough approximation \( x_1 \). Also note if the given values for \( x_1 \) are good choices to start Newton’s Method. (8P)

(b) Use Newton’s Method to find an approximation \( x_2 \) for a root of \( f(x) = x^2 - 2 \), given the rough approximation \( x_1 = 2 \). (5P)
5. Find the absolute minimum and maximum values of \( f(x) = x - 2\sqrt{x} \) on the interval \( 0 \leq x \leq 9 \). (12P)

6. (a) State the Mean Value Theorem. (3P)

(b)* Check if the following is possible: A differentiable function \( f(x) \) on \([0, 1]\) has the property that \( f(0) = 1 \), \( f(1) = 3 \) and \( f'(x) \leq 1 \) for all \( x \) in \([0, 1]\). If this is possible, find such a function. If it is not possible, explain why such a function does not exist. (5P*)
7. Let $f(x) = x^3 - 3x^2$. (total: 20P)
   (a) Find the intervals in which $f$ is increasing/decreasing. (4P)
   (b) Find all local extreme values of $f$. (3P)
   (c) Determine the intervals where $f$ is concave up, concave down. (4P)
   (d) Find the points of inflection of $f$. (3P)
   (e) Sketch the graph of $f$. (6P)
8. A farmer wants to fence off a rectangular field with an area of 800 m$^2$. One side of the field borders to a river where no fence is needed. She wants to buy as little fence as possible. How much fence is needed? (10P)