TEST II

No calculators are allowed!

PART I

Part I consists of 6 questions. Clearly write your answer (only) in the space provided after each question. You do not need not to show your work for this part of the test. No partial credit is awarded for this part of the test!

Each question is worth 5 points.

Question 1

Differentiate the function $f(x) = (x - 2)(2x + 3)$. (Simplify your answer!)

Answer: 

Question 2

For what values of $x$ does the graph of the function $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 + 1$ have a horizontal tangent line? (You should find two values in all!)

Answer: 
Question 3

Suppose \( h(x) = f(x)g(x) \) where \( f(3) = 4, g(3) = 2, f'(3) = 6, \) and \( g'(3) = 5 \). Find the numerical value of \( h'(3) \).

Answer: .................

Question 4

Let \( h(x) = \frac{f(x)}{\cos x} \) where \( g(x) \) is a differentiable function on the real line with \( f'(0) = -2 \). Find the numerical value of \( h'(0) \).

Answer: .................

Question 5

Find the derivative of the function \( f(x) = (x^2 + x + 3)^2 \).

Answer: .................

Question 6

Use implicit differentiation to find \( \frac{dy}{dx} \) if it is known that \( x^2 = 4 - 4y^2 \).

Answer: .................
Part II consists of 5 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit.

Problem 1

The equation of a motion of a particle is given by

\[ s(t) = t^4 - 4t \]

where \( s \) is in miles and \( t \) is in seconds.

(a) Find the velocity of the particle as a function of \( t \).

(b) Find the acceleration of the particle as a function of \( t \).

(c) Find the acceleration of the particle after 2 seconds.

(d) Find the acceleration of the particle when the velocity is zero.
Problem 2

For which (numerical) values of $a$ and $b$ is the line $y - 4x = b$ tangent to the curve $y = ax^2$ when $x = 1$. 
Problem 3

(a) Find the derivative of the function

\[ g(x) = (x^2 + 1) \sqrt[3]{x^2 + 4}. \]

(Simplify and write your answer as one expression!)

(b) Find the derivative of the function

\[ f(t) = \frac{2t}{4 + t^2}. \]

(Simplify and write your answer as one fraction!)
Problem 4

(a) Find the derivative of the function \( f(x) = \sin(x \tan x) \).

(b) Use implicit differentiation to find an equation of the tangent line to the curve

\[
x^2 + x \sin y + xy = 1
\]

at the point (1, 0).
Problem 5

Two people start walking from the same point. One walks north at 4 mi/h and the other walks east at 3 mi/h. At what rate is the distance between these people increasing 2 hours later. (Show your work!)

(Hint: The distance each has walked in 2 hours might prove useful in your calculations!)
SCRATCH PAPER