Part I

Part I consists of 10 questions, each worth 5 points. Clearly show your work for each of the problems listed.

(1) Let \( f(x) = 4x^3 - 12x + 2 \). Find all local max/min of \( f(x) \). State both \( x \) and \( y \) coordinates.
   \[ \text{Answer: local max at } (-1, 10) \text{ and local min at } (1, -6). \]

(2) Find the absolute max/min of \( f(x) = 10 - x^2 \) on the interval \([-1, 2]\). Give both \( x \) and \( y \)-coordinates and justify your answer.
   \[ \text{Answer: absolute max at } (0, 10) \text{ and absolute min at } (2, -6). \]

(3) Find two positive numbers whose product is 100 and whose sum is minimal. (You must justify your answer.)
   \[ \text{Answer: } 10 \text{ and } 10. \]
(4) Let \( f'(x) = (x-2)^2(x-1)(x+1) \). **Note that you are already given the derivative \( f'(x) \).** Find all critical points, where \( f(x) \) is increasing and decreasing, and also find the \( x \)-coordinate(s) of all local max/min.  

**Answer:** increasing on \((-\infty, -1)\) and \((1, \infty)\), decreasing on \((-1, 1)\). Local min at \(x = 1\), local max at \(x = -1\).  

(5) If \( f''(x) = (x-1)^2(x+3) \) find where \( f(x) \) is concave up and where it is concave down. Also find all points of inflection.  

**Note that you are given \( f''(x) \)!**  
**Answer:** concave up on \((-3, \infty)\), concave down on \((-\infty, -3)\), inflection point at \(x = -3\).  

(6) Find the most general anti-derivative of \( \frac{3-x+4\sqrt{x}}{x^4} \).  

**Answer:** \(-\frac{3}{2}x^{-2} + x^{-1} - \frac{8}{3}x^{-3/2} + C\).
(7) Find the most general anti-derivative of $\cos(x) - \frac{1}{x}$.
Answer: $\sin x - \ln |x| + C$.

(8) Find all asymptotes of the function $\frac{x^3+5}{x(x-2)(x+1)}$.
Answer: vertical: $x = 0, x = 2, x = -1$ and horizontal $y = 1$.

(9) If the acceleration is given by $a(t) = 6t$, $v(0) = 2$ and $s(0) = 1$, find an expression for the position $s(t)$.
Answer: $s(t) = t^3 + 2t + 1$.

(10) If $f(x) = x^3$ find the number $c$ that satisfies the conclusion of the mean value theorem on the interval $[0, 2]$.
Answer: $c = 2/\sqrt{3}$. 
Part II

Part II consists of 3 problems; the number of points for each part are indicated by [x pts]. You must show the relevant steps (as we did in class) and justify your answer to earn credit. Simplify your answer when possible.

(1) [15 pts] Find the absolute max/min of the function \( f(x) = (x^2 - 1)^3 \) on the interval \([-2, 3]\).
Answer: absolute min \((0, -1)\), absolute max \((3, 512)\).

(2) Given the function \( f(x) = \frac{x^2 - 9}{x^2 - 1} \).
(a) [2 pts] Find the \( x \) and \( y \) intercepts of the function.
Answer: \( x = \pm 3 \) and \( y = 9 \).

(b) [3 pts] Find all asymptotes.
Answer: vertical \( x = 1 \) and \( x = -1 \), horizontal \( y = 1 \).
(c) [4 pts] Find the open intervals where \( f(x) \) is increasing and the open intervals where \( f(x) \) is decreasing.
Answer: increasing on \((0, 1)\) and \((1, \infty)\), decreasing on \((-\infty, -1)\) and \((-1, 0)\).

(d) [2 pts] Find the local maximum and local minimum value(s) of \( f(x) \). (Be sure to give the \( x \) and \( y \) coordinate of each of them).
Answer: local min at \((0, 9)\), no local max.

(e) [2 pts] Find all open intervals where the graph of \( f(x) \) is concave up and all open intervals where the graph is concave down.
Answer: concave up on \((-1, 1)\), concave down on \((-\infty, -1)\) and \((1, \infty)\).

(f) [2 pts] Find all points of inflection (be sure to give the \( x \) and \( y \) coordinate of each point).
Answer: \( x = 1 \) and \( x = -1 \).

Correct answer: no inflection points.

(g) [6 pts] Use the above information to graph the function on the next page. Indicate all relevant information in the graph.
Put the graph of Problem 2 on this page.
(3) **[14 pts]** A drilling rig in the ocean is 10 mi of shore. A refinery is located along the coast 15 mi away from the point on the shore closest to the rig. If under water pipe lines cost $5 per mi and land-based pipe costs $4 per mi, what is the least expensive way to run the line.

Partial answer: The total cost of the pipe is

\[ f(x) = 4(15 - x) + 5\sqrt{10^2 + x^2} \]

you need to find \( x \) where this function is a minimum.