TEST II

No calculators are allowed!

PART I

Part I consists of 6 questions. Clearly write your answer (only) in the space provided after each question. You do not need not to show your work for this part of the test. No partial credit is awarded for this part of the test!

Each question is worth 5 points.

Question 1

Differentiate the function \( f(x) = (x - 3)\cos(x) \).

Answer: .....................

Question 2

For what values of \( x \) does the graph of the function \( f(x) = \frac{1}{5}x^5 - \frac{1}{4}x^4 \) have a horizontal tangent line? (You should find two values in all!)

Answer: .....................
Question 3

Suppose $h(x) = f(x)g(x)$ where $f(1) = 0$, $g(1) = 1$, $f'(1) = -1$ and $g'(1) = 5$. Find the numerical value of $h'(1)$. [Your answer should be a number!]

Answer: .....................

Question 4

Let $h(x) = \frac{f(x)}{\sin(x)}$ where $g(x)$ is a differentiable function on the real line with $f'(\pi/2) = -2$. Find the numerical value of $h'(\pi/2)$. [Your answer should be a number!]

Answer: .....................

Question 5

Find the derivative of the function $f(x) = \sqrt[3]{x^3 + x - 1}$.

Answer: .....................

Question 6

Use implicit differentiation to find $\frac{dy}{dx}$ if it is known that $3x^2 - 2y^3 = 2$.

Answer: .....................
Each problem is worth 14 points.

Part II consists of 5 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit.

**Problem 1**

The equation of a motion of a particle is given by

\[ s(t) = t^3 - 9t^2 + 15t + 1 \]

where \( s \) is in miles and \( t \) is in seconds.

(a) Find the velocity of the particle as a function of \( t \).

(b) Find the acceleration of the particle as a function of \( t \).

(c) Find the acceleration of the particle after 2 seconds.

(d) Find the acceleration of the particle when the velocity is zero. (You should find two values in all!!)
Problem 2

This problem has two separate questions! Answer each question.

(a) For which (numerical) values of $a$ and $b$ is the line $y - 2x = b$ tangent to the curve $y = ax^2$ when $x = 1$.

(b) Differentiate the function

$$y = \frac{x + \cos(x)}{1 + \sin(x)}.$$

(Simplify and write your answer as one single fraction!)
Problem 3

(a) Find the derivative of the function

\[ g(x) = (x + 1) \sqrt{x^2 + 2}. \]

(Simplify and write your answer as one expression!)

(b) Find the derivative of the function

\[ f(t) = \frac{t - 1}{t + 1}. \]

(Simplify and write your answer as one fraction!)
Problem 4

(a) Find the derivative of the function \( f(x) = \tan(x \sin x) \).

(b) Use implicit differentiation to find an equation of the tangent line to the curve

\[ x^2 + xy + y^2 = 1 \]

at the point \((1, 0)\).
**Problem 5**

A car travels south at 40 mi/h and another travels west at 30 mi/h. They are both headed for the intersection of the two roads. At what rate are the cars approaching each other when the first car is 0.2 mi and the second one is 0.4 mi away from the intersection? (Show your work!)
SCRATCH PAPER