Instruction: Answer the questions in the space provided. Use the scratch paper provided if needed. Please keep your answers neat, complete but brief, and to the point.

Question 1
Question 2
Question 3
Question 4
Question 5
Question 6
Total

Please do not write in this box
QUESTION 1. Define:

\[ f(x) = \int_{0}^{x} \cos(t^2) \, dt, \quad 0 < x < \sqrt{\pi}. \]

Determine the interval(s) where \( f \) is increasing.
QUESTION 2. Find the integral:

\[ \int \frac{dx}{\sqrt{4 + x^2}}. \]

Show all the steps in your derivation. 

*Hint:* Substitute \( x = 2 \sinh u. \)
QUESTION 3. Compute:

\[ \int_{0}^{1} \frac{dx}{x^2 - 4}. \]
QUESTION 4. Find the integral:

\[ \int \cos^3 x \sin^2 x \, dx. \]
QUESTION 5. The midpoint method $M_n$ is used to approximate the following integral:

$$\int_0^1 e^{x^3} \, dx.$$ 

How large should one choose $n$ in order to guarantee the error is less than $10^{-6}$?

*Hint:* Recall that the error in the midpoint method can be estimated by:

$$|E_M| \leq \frac{K(b-a)^3}{24n^2}.$$
QUESTION 6. Determine whether the following improper integral converges:

\[ \int_{1}^{\infty} \frac{1}{\sqrt{x^3 + 1}} \, dx. \]