Calculus 2
MA126-6B
Midterm Examination 2
Tuesday, November 18, 2003

Instruction: Answer the questions in the space provided. Use the scratch paper provided if needed. Please keep your answers neat, complete but brief, and to the point.

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*Please do not write in this box*
QUESTION 1. Find the volume of the solid of revolution obtained by rotating the area under the curve

\[ y = x \cos x, \quad 0 \leq x \leq \pi/2, \]

about the y-axis:

*Hint:* Use cylindrical shells.
QUESTION 2. Find the area bounded by the two curves:

\[ y^2 = x + 2, \quad y = |x|. \]
QUESTION 3. Find the arclength of the curve:

$$x = y^{3/2}, \quad 0 \leq y \leq 1.$$
QUESTION 4. Check that the function:

\[ f(x) = \begin{cases} 
6x(1 - x) & \text{if } 0 \leq x \leq 1 \\ 
0 & \text{otherwise} 
\end{cases} \]

is a probability density function. Find the mean, standard deviation, and median.
QUESTION 5. Find the limit:

$$\lim_{n \to \infty} \frac{n \cos n}{n^2 + 1}.$$ 

Justify your answer.
QUESTION 6. Determine whether the following series converges:

\[ \sum_{n=1}^{\infty} \ln \left( 1 + \frac{1}{n} \right). \]

Justify your answer.
QUESTION 7. Determine whether the following series converges, converges absolutely, or converges conditionally:

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}.$$ 

*Hint: Use the integral test.*
QUESTION 8. Find the Maclaurin series for the function:

\[ f = \frac{1}{(1-x)^2}. \]

Determine the interval of convergence.

*Hint:* \(1/(1-x)^2\) is the derivative of \(1/(1-x)\).
QUESTION 9. Let $f(x) = x\arctan x$. Find $f^{(126)}(0)$, the 126\textsuperscript{th} derivative of $f$ at $x = 0$.

Hint: Find the Maclaurin series of $f$, and use Taylor’s formula for the 126\textsuperscript{th} coefficient.
QUESTION 10. Check that the series

\[ \sum_{n=0}^{\infty} \frac{1}{(2n)!} \]

converges, and find its sum.

*Hint: Find the Maclaurin series of \( \cosh x = \frac{e^x + e^{-x}}{2} \).*