1. Evaluate the following definite, indefinite and improper integrals (5P+5P+5P+5P+5P):
   (a) \( \int e^x \sin(e^x) \, dx \)

   (b) \( \int xe^{3x} \, dx \)
(c) \[ \int \frac{1}{x(1 + x)} \, dx \]

(d) \[ \int_{1}^{e} \frac{(\ln x)^3}{x} \, dx \]

(e) \[ \int_{0}^{\infty} \frac{x}{(1 + x^2)^2} \, dx \]
2. (a) Find the area enclosed between the two curves \( y = x^2 \) and \( y = x \). (4P)

(b) A solid is generated by revolving the region between the curves \( y = x^2 \) and \( y = x \) about the \( x \)-axis. Find its volume. (4P)

3. Find the limits of the following sequences (3P+3P+3P):

(a) \( \lim_{n \to \infty} \frac{n^3}{1 - 2n^3} \)

(b) \( \lim_{n \to \infty} \frac{(-1)^n}{n} \)

(c) \( \lim_{n \to \infty} \frac{e^n}{1 + e^n} \)
4. Do the following series converge or diverge? Find the sum of those which converge (4P+4P).

(a) \( \sum_{n=0}^{\infty} 3 \left( \frac{1}{\sqrt{2}} \right)^n \)

(b) \( \sum_{n=1}^{\infty} \frac{1}{3} (\sqrt{2})^n \)

5. Do the following series converge or diverge? Justify your answer by referring to the tests which were used (4P+4P+4P+4P).

(a) \( \sum_{n=1}^{\infty} \frac{1}{n^2 + \ln n} \)

(b) \( \sum_{n=1}^{\infty} \frac{1}{\sqrt{n} + 3} \)
(c) \[ \sum_{n=1}^{\infty} \frac{e^n}{e^n + 1000} \]

(d) \[ \sum_{n=2}^{\infty} (-1)^n \frac{1}{\ln n} \]

6. Find the radius and interval of convergence for the power series (6P)

\[ \sum_{n=1}^{\infty} \frac{3^n x^n}{n^2} \]
7. Find the second order Taylor polynomial $T_2(x)$ for $f(x) = \sqrt{x}$ at $a = 1$. (4P)

8. Let $\mathbf{a} = \langle 1, -1, 1 \rangle$, $\mathbf{b} = \langle 3, 2, 1 \rangle$ and $\mathbf{c} = \langle 2, 2, 0 \rangle$. Find
   (a) the cosine of the angle between $\mathbf{a}$ and $\mathbf{b}$ (3P),

   (b) the vector projection of $\mathbf{b}$ onto $\mathbf{a}$ (3P),

   (c) The area of the parallelogram determined by $\mathbf{a}$ and $\mathbf{b}$ (3P),

   (d) The volume of the parallelepiped (skew box) determined by $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ (3P).
9. Find an equation for the plane which passes through the points \( P(-1, -1, -1), \)
\( Q(1, 1, 1) \) and \( R(0, 2, 3) \). (6P)

10. Find parametric equations for the line of intersection of the two planes \( x-y+z = 2 \)
and \( x+y-2z = 1 \). (6P)
11*. Find a positive number $c$ such that $\sum_{n=1}^{\infty} (1 + c)^{-n} = 2$. (5P*)

12*. Use the Fundamental Theorem of Calculus to show the following: If the function $f$ is continuous and if $\int_{-x}^{x} f(t) \, dt = 0$ for every $x > 0$, then $f$ is odd (meaning that $f(-x) = -f(x)$ for every $x > 0$). (5P*)