TEST 1
Duration 70min;

Make sure to show all your work and underline the final results of each problem. Write your name on this sheet and use it as a cover page when you turn in your work. Do not write your results on this paper. Good luck!

1. The graph of $f$ consists of two straight lines and a semi circle. Use it to evaluate each integral.

   (a) $\int_{3}^{4} f(x) \, dx$, (b) $\int_{-1}^{2} f(x) \, dx$, (c) $\int_{0}^{4} f(x) \, dx$

2. (a) Use the properties of integrals to verify that

   $$\int_{1}^{\frac{1}{x+2}} \frac{1}{\ln(x) + 2} \, dx \leq 1$$

   (b) Derive a good lower bound for integral in a similar way as the upper bound is derived. (E.g. $-1$ is a correct lower bound but not good enough.)

3. Write out the form of the partial fraction expansion of the function. Do not determine the numerical values of the coefficients.

   (a) $\frac{2x - 7}{(x - 1)^2(x + 1)}$, (b) $\frac{1 + 3x - x^2}{(x^2 - 14x + 50)x}$

4. Evaluate the following integrals

   (a) $\int_{1}^{2} u^3 \, du$
5. Find the derivative of the function

\[ g(x) = \int_0^{\sin(x)} \frac{2}{t^2 + \ln(t + 2)} \, dt \]

**Bonus.** Prove the following statement. If

\[ \int_{-x}^{x} f(t) \, dt = 0 \quad \text{for all } x > 0 \]

then \( f \) is an odd function: \( f(-x) = -f(x) \).