TEST 1
Duration 70min;

Make sure to show all your work and underline the final results of each problem. Write your name on this sheet and use it as a cover page when you turn in your work. Do not write your results on this paper. Good luck!

1. The graph of \( f \) consists of two straight lines and a semi circle. Use it to evaluate each integral.
   \[ (a) \int_{3}^{4} f(x) \, dx, \quad (b) \int_{0}^{2} f(x) \, dx, \quad (c) \int_{1}^{4} f(x) \, dx \]

2. (a) Use the properties of integrals to verify that
   \[ \int_{1}^{3} \frac{1}{\ln(x) + 2} \, dx \leq 1 \]
   (b) Derive a good lower bound for integral in a similar way as the upper bound is derived.
   (E.g. \(-1\) is a correct lower bound but not good enough.)

3. Write out the form of the partial fraction expansion of the function. Do not determine the numerical values of the coefficients.
   \[ (a) \frac{4x - 1}{(x + 1)^2(x - 3)} \quad (b) \frac{1 + 5x - x^2}{(x^2 + 2x + 6)(x - 1)} \]

4. Evaluate the following integrals
   \[ (a) \int_{1}^{2} s^3 \, ds \]

1
(b) \[ \int \frac{2 - 3u}{\sqrt{u}} \, du \]

(c) \[ \int \frac{1}{3x - 7} \, dx \]

(d) \[ \int_{-3}^{3} \frac{\sin(x)x^6}{1 + x^4} \, dx \]

(e) \[ \int (\sin x)^4(\cos x)^3 \, dx \]

(f) \[ \int x^2(1 - x^3)^7 \, dx \]

(g) \[ \int t^{1/2} \ln(t) \, dt \]

(h) \[ \int \frac{x^4}{x^2 + 1} \, dx \]

(i) \[ \int \frac{1}{x^2 - 1} \, dx \]

5. Find the derivative of the function

\[ g(x) = \int_{0}^{1/x} \frac{3}{t^2 + \ln(t + 2)} \, dt \]

**Bonus.** Prove the following statement. If

\[ \int_{-x}^{x} f(t) \, dt = 0 \quad \text{for all } x > 0 \]

then \( f \) is an odd function: \( f(-x) = -f(x) \).