Math 126 TEST III

Do not use any books or notes. You can use a calculator, but not graphing calculator. If you use a calculator, leave your results in exact form instead of decimal form. Show all work for full credit.

1. Determine whether the SEQUENCE converges or diverges. If it is convergent, find the LIMIT. (15 points)

   (a) \( a_n = \frac{n+1}{3n-1} \)  
   (b) \( a_n = \frac{n}{1+\sqrt{n}} \)  
   (c) \( a_n = \frac{\ln(n^3)}{n} \)

2. Determine whether the SERIES is convergent or divergent. If it is convergent, find the SUM. (18 points)

   (a) \( \sum_{n=1}^{\infty} \left( \frac{1}{2^{n-1}} + \frac{2}{3^{n-1}} \right) \)  
   (b) \( \sum_{n=1}^{\infty} \frac{1}{5 + 2^{-n}} \)  
   (c) \( \sum_{n=1}^{\infty} \left[ \sin \left( \frac{1}{n} \right) - \sin \left( \frac{1}{n+1} \right) \right] \)
3. Determine whether the **series** is convergent or divergent. You do NOT need to find the sum. But you need to support your claim by appropriate work. (16 points)

   (a) \( \sum_{n=1}^{\infty} \frac{n+1}{n^2} \)  
   (b) \( \sum_{n=1}^{\infty} \frac{\sin^2 n}{n\sqrt{n}} \)

4. For each series determine whether it is convergent or divergent, and whether it is **absolutely convergent**. Support your claim by appropriate work. (16 points)

   (a) \( \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}} \)  
   (b) \( \sum_{n=0}^{\infty} \frac{(-3)^n}{n!} \)
5. Find the RADIUS of convergence and INTERVAL of convergence of the power series. (20 points)

(a) \[ \sum_{n=1}^{\infty} \frac{x^n}{n^2} \]

(b) \[ \sum_{n=0}^{\infty} \frac{n^2 x^n}{10^n} \]
6. Find a **POWER SERIES** representation for the function $f(x) = \frac{x}{1 - 2x}$ and then determine the **RADIUS** of convergence. (8 points)

7. Find the Maclaurin series for $f(x) = \cos x$ using the definition $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$. (7 points)