1. How large do we have to choose $n$ so that the approximations $T_n$ and $M_n$ to the integral

$$\int_0^{\sqrt{\pi}} \sin(x^2)dx$$

are accurate within 0.0001? 

10 points
2. Calculate the improper integral

\[ \int_{1}^{\infty} 2^{-x} \, dx. \]

10 points
3. Determine if the improper integral

\[ \int_0^\infty \frac{1}{(x^4 + 2)^{1/3}} \, dx \]

is convergent or divergent. Explain!  

10 points
4. Find the number $a$ such that the line $x = a$ bisects the area under the curve $y = \frac{1}{x}$, $1 \leq x \leq 1.44$. Also, find the number $b$ such that the line $y = b$ bisects the above area.

10 points
5. Consider the region $D$ bounded by the curves $y = x^{2/3}$, $x = 8$, $y = 0$. Find the volume of the solid $S$ that can be obtained by revolving $D$ about the $y$-axis.

10 points
6. Find the exact length of the curve \( x = e^t + e^{-t}, \ y = 2t, \ 0 \leq t \leq 1. \) 
10 points
7. Find the centroid of the lamina (with constant density) bounded by the curves $y = 1/x, y = 0, x = 1, x = 4.$
8. Let \( a_n = \cos \left( \frac{\pi}{2n} \right) \). Check the sequence \( \{a_n\} \) for monotonicity and boundedness. Is it convergent? Explain everything!  

10 points
9. The sequence \( \{a_n\} \) is defined recursively by \( a_1 = \sqrt{6} \), \( a_{n+1} = \sqrt{6 + a_n} \) for \( n = 1, 2, \ldots \). Is it convergent? Explain! Find \( \lim_{n \to \infty} a_n \).  

10 points
10. Find the values of $x$ for which the series

$$\sum_{n=0}^{\infty} 3^n (x + 2)^{n+1}$$

converges. Find the sum of the series for those values of $x$. 10 points