TEST II

Closed book test – No calculators are permitted!

PART I - Basic Skills

Each question is worth 5 points.

Part I consists of 6 questions. Clearly write your answer (only) in the space provided after each question. You do not need to show your work for this part of the test. No partial credit is awarded for this part of the test!

Question 1

Find the derivative of the function \( g(x) = \int_{1}^{x} \ln(t^5) \, dt \).

Answer: ......................

Question 2

If \( \int_{0}^{7} f(x) \, dx = 10 \) and \( \int_{7}^{9} f(x) \, dx = -5 \), find the numerical value of \( \int_{0}^{9} f(x) \, dx \).

Answer: ......................
Question 3

Evaluate the definite integral
\[ \int_1^e \frac{1}{2x} \, dx \]
(Your answer must be a real number!)

Answer: ..................

Question 4

Evaluate the indefinite integral
\[ \int (3x^2 + 2x - 1) \, dx. \]

Answer: ..................

Question 5

Evaluate the definite integral
\[ \int_0^1 xe^x \, dx. \]
(Your answer must be a real number!)

Answer: ..................

Question 6

Evaluate the indefinite integral
\[ \int \sin^2 x \cos x \, dx. \]

Answer: ..................
PART II - Problem Solving skills

Each problem is worth 14 points.

Part II consists of 5 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit.

Problem 1

Consider the function $f(x) = 1 + x^2$ on the interval $[-1, 1]$.

(a) Find the average value, $f_{\text{ave}}$, of the function $f$ on the given interval.

(b) Find the numerical value(s) of $c$ such that $f(c) = f_{\text{ave}}$. [Hint: You should find two values in all!]
Problem 2

The velocity function (in miles per second) of an object moving along a line is given by

\[ v(t) = t^2 + 2t - 3, \quad 0 \leq t \leq 2. \]

(a) Find the displacement (in miles) of the object during the given time interval \(0 \leq t \leq 2\).

(b) Find the distance (in miles) traveled by the object during the given time interval \(0 \leq t \leq 2\).

(Simplify and express your answer as a fraction if need be!)
Problem 3

This problem has two separate questions (a) and (b). Answer each question.

(a) Evaluate the definite integral

$$\int_{-3}^{3} \sqrt{9 - x^2} \, dx$$

by interpreting it in terms of areas.

(Hint: Note that the integral represents the area of a portion of a well-known shape in the plane!)

(b) Evaluate the definite integral

$$\int_{-1}^{1} \sin(2\pi t) \, dt.$$  

(Your answer must be a number! Justify your answer of course.)
Problem 4

This problem has two separate questions (a) and (b). Answer each question.

(a) Evaluate the definite integral
\[ \int_1^2 \frac{y + 3y^5}{y^3} \, dy. \]

(b) Evaluate the indefinite integral \( \int x \sin x \, dx. \)
Problem 5

This problem has two separate questions (a) and (b). Answer each question.

(a) Evaluate the indefinite integral  
\[ \int x^2 \ln x \, dx. \]

(b) Find the exact area of the region between the graph of the function \( f(x) = 2xe^{-x^2} \) and the x-axis when \( 0 \leq x \leq 3 \). (Hint: Substitution method might prove useful here!)