Show all your work and give reasons for your answers. Good luck!

Part I

Each problem in part I is worth 5 points; Show your work!!

(1) Find the angle between the vectors \( \vec{a} = <0, 1, 2> \) and \( \vec{b} = <-1, 2, -3> \). (You may express your answer in terms of \( \arccos \).)

(2) Find the equation of the plane perpendicular to the line \( x = 2 + t, y = 1 - 2t \) and \( z = 1 - 3t \) which passes through the point \((-1, -1, -3)\).

(3) Find the area of the parallelogram spanned by the vectors \( <1, 0, 1> \) and \( <-1, 2, 1> \).

(4) If \( \vec{a} = <-2, -1, 3> \) and \( \vec{b} = <2, 1, 3> \) find the component \( \text{com}_b(\vec{a}) \).
(5) if $\vec{u} = <1, 0, 1>$ and $\vec{v} = <2, 5, -2>$ is $\vec{u}$ perpendicular to $\vec{v}$? (You must justify your answer.)

(6) If $\vec{r}(t) = <\sin(t), t^3, e^t>$, find $\lim_{t \to \pi} \vec{r}(t)$.

(7) If $\vec{r}(t) = <e^t, t\cos(t), t^2>$ find the derivative $\vec{r}(t)'$.

(8) If $\vec{r}(t) = <e^t, t\cos(t), t^2>$, find the unit tangent vector $T(t)$.

(9) Find the distance between the planes $2x + y - z = 3$ and $4x + 2y - 2z = 10$.

(10) Are the lines $\frac{x+1}{1} = \frac{y+2}{5} = \frac{z-2}{-2}$ and $\frac{x+4}{2} = \frac{y-7}{10} = \frac{z+11}{-4}$ parallel? (You must justify your answer.)
(11) Are the vectors $<1, 0, 2>$, $<2, 3, 1>$ and $<1, 3, -1>$ coplanar (You must justify your answer!)
Part II

1. (a) **[5 points]** Find the area of the triangle with vertices \( P = (1, 2, 1) \), \( Q = (2, 2, 3) \) and \( R = (3, 1, 1) \).

   (b) **[5 points]** Determine if the vectors \( \langle 1, 0, 1 \rangle \), \( \langle 2, -1, 1 \rangle \) and \( \langle -1, 2, 1 \rangle \) are coplanar. (You must justify your answer.)
(2) [15 points] Given the lines:

\[ \ell_1 = \begin{cases} 
  x = -1 + 2t \\ 
  y = 1 + t \\ 
  z = 4 - t 
\end{cases} \quad \text{and} \quad \ell_2 = \begin{cases} 
  x = -1 + t \\ 
  y = 2 + 3t \\ 
  z = -1 - 2t 
\end{cases} \]

determine if they are skew or not. If they are skew, find their distance. If not, find the point of intersection.
(3) [10 points] Find the line of intersection of the planes
\[ x - 2y + 3z = 1 \] and \[ 2x + y - z = 4. \]

(4) Let \( \vec{r}(t) = < t \sin(t), (t^2 + 1)^5, \ln(t) > \) be the position of a fly at time \( t \), find
(a) [5 points] The velocity vector \( \vec{v}(t) \) at time \( t = \pi \).

(b) [5 points] The speed at time \( t = \pi \).
(c) [5 points] The unit tangent vector $\vec{T}(t)$ at time $t = \pi$.

(5) [Bonus: 5 points] Assume that $|\vec{r}(t)| = c$ is constant show that $\vec{r}(t)$ is perpendicular to $\vec{r}(t)'$. (Hint use the fact that $\vec{r}(t) \cdot \vec{r}(t) = c^2$ is constant and, hence $0 = \frac{d}{dt} [\vec{r}(t) \cdot \vec{r}(t)]$.) Do you see a geometric interpretation of this fact?