Question 1

If \( \int_{-1}^{5} f(x) \, dx = 10 \) and \( \int_{1}^{5} f(x) \, dx = -12 \), find the numerical value of \( \int_{-1}^{1} f(x) \, dx \).

Answer: .....................

Question 2

Find the derivative of the function \( g(x) = \int_{-2}^{x} t^2 \sin(3t) \, dt \).

Answer: .....................
Question 3

Evaluate the definite integral

\[ \int_{e^{34}}^{e^{35}} \frac{2}{x} \, dx \]

(Your answer must be a real number!)

Answer: ........................

Question 4

Evaluate the indefinite integral

\[ \int \frac{e^x}{e^x + 1} \, dx \]

Answer: ........................

Question 5

Evaluate the indefinite integral

\[ \int x \cos(x) \, dx \]

Answer: ........................

Question 6

Evaluate the indefinite integral

\[ \int \frac{x}{x + 5} \, dx \]

Answer: ........................
Part II

Each problem is worth 14 points.

Part II consists of 5 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit - no credit for unsubstantiated answers!

Problem 1

Consider the function $f(x) = (x - 3)^2$ on the interval $[2, 5]$.

(a) Find the average value, $f_{\text{ave}}$, of the function $f$ on the given interval. (Your answer must be a real number!)

(b) Find the numerical value(s) of $c$ such that $f(c) = f_{\text{ave}}$. 
Problem 2

The velocity function (in meters per second) of an object moving along a line is given by

\[ v(t) = t - 2, \quad 0 \leq t \leq 4. \]

(a) Find the displacement (in meters) of the object during the time interval \( 0 \leq t \leq 4 \).

(b) Find the distance (in meters) traveled by the object during the time interval \( 0 \leq t \leq 4 \).
Problem 3

This problem has two separate questions (a) and (b). Answer each question.

(a) Evaluate the definite integrals

\[ \int_{-6}^{0} \sqrt{36 - x^2} \, dx \quad \text{and} \quad \int_{-2}^{2} |x| \, dx \]

by interpreting them in terms of areas.

(b) Evaluate the definite integral

\[ \int_{1}^{4} \frac{3x - 4}{\sqrt{x}} \, dx. \]

(Your answer must be a real number!)
Problem 4

This problem has two separate questions (a) and (b). Answer each question.

(a) Evaluate the definite integral \( \int_{e^{49}}^{e^{16}} \frac{1}{x \sqrt{\ln(x)}} \, dx \). (Your answer must be a real number!)

(b) Evaluate the indefinite integral \( \int \sin^4(x) \cos^3(x) \, dx \).
Problem 5

This problem has two separate questions (a) and (b). Answer each question.

(a) Evaluate the indefinite integral \( \int x^2 e^x \, dx \).

(b) Evaluate the indefinite integral \( \int \frac{x - 11}{(x + 1)(x - 2)} \, dx \).
SCRATCH PAPER
(Scratch paper will not be graded!)
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