Calculus II, Exam IV, Spring 2012

Name: ____________________________

Student signature: _______________________

Show all your work and give reasons for your answers. Good luck!

(1) (6 points) Find the sum of the series \( \sum_{n=0}^{\infty} \frac{1}{3^n} = 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots \)

(2) (10 points) Test the following series for absolute or conditional convergence, or divergence:

\[
\sum_{n=1}^{\infty} \frac{(-1)^n n^3}{\sqrt{n^6 + n^5}}
\]
(3) (12 points) Find the interval and radius of convergence for \( \sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^n}{n^2} \).

(4) (18 points) Find the MacLaurin series and state the radius of convergence for \( f(x) = \ln(1 + 3x) \).
(5) (18 points) Find the MacLaurin series and state the radius of convergence for \( g(x) = \frac{x^2}{7+x} \).
(6) (18 points) Use the MacLaurin series to evaluate $\sin(1/10)$ with an error less than $10^{-5}$.

(7) (18 points) Use the MacLaurin series to approximate $\int_0^{1/10} \sin(x^2) \, dx$ with an error less than $10^{-5}$. 