1. Find the velocity and position vectors of a particle that has the given acceleration and the given initial velocity and position:

\[ a(t) = 2i + 6t \mathbf{j}, \quad v(0) = -2i + k, \quad r(0) = \mathbf{j} + 5k. \]

Answers:

\[ v(t) = (2t - 2)i + 3t^2 \mathbf{j} + k \]

and

\[ r(t) = (t^2 - 2t)i + (t^3 + 1)\mathbf{j} + (t + 5)k \]
2. The position of a moving particle is given by

\[ \mathbf{r}(t) = \langle t, t^3, \ln t \rangle \]

(a) Find velocity vector, speed, and acceleration vector at the point where \( t = 1 \);
(b) Find the curvature of the particle’s trajectory at the point where \( t = 1 \);
(c) Find the tangential and normal components of the acceleration vector at the point where \( t = 1 \).

Answers:

\[ \mathbf{v}(1) = \langle 1, 3, 1 \rangle, \quad v(1) = \sqrt{11}, \]
\[ \mathbf{a}(1) = \langle 0, 6, -1 \rangle \]

and

\[ \kappa(1) = \frac{\sqrt{118}}{11^{3/2}}, \quad a_T(1) = \frac{17}{\sqrt{11}}, \quad a_N(1) = \frac{\sqrt{118}}{\sqrt{11}} \]
3. Find the length of the curve:

\[ \mathbf{r}(t) = \langle \cos 3t, \sin 3t, 2(t - 1)^{3/2} \rangle \quad 1 \leq t \leq 4. \]

Answer:

\[
L = \int_1^4 \sqrt{9 \sin^2 3t + 9 \cos^2 3t + 9(t - 1)} \, dt = \int_1^4 3\sqrt{t} \, dt = 14.
\]
4. A vector function is given:

\[ \mathbf{r}(t) = 4t \mathbf{i} + 3 \cos t \mathbf{j} + 3 \sin t \mathbf{k}. \]

(a) Find the unit tangent vector \( \mathbf{T} \), the unit normal vector \( \mathbf{N} \), and the unit binormal vector \( \mathbf{B} \).

(Bonus) Find equations of the normal plane and the osculating plane at the point where \( t = \pi/2 \).

Answer:

\[
\mathbf{T}(t) = \langle \frac{4}{5}, -\frac{3}{5} \sin t, \frac{3}{5} \cos t \rangle \\
\mathbf{N}(t) = \langle 0, -\cos t, -\sin t \rangle \\
\mathbf{B}(t) = \langle \frac{3}{5}, \frac{4}{5} \sin t, -\frac{4}{5} \cos t \rangle 
\]

Normal plane:

\[ 4x - 3y = 8\pi \]

Osculating plane:

\[ 3x + 4y = 6\pi \]
5. Find parametric equations for the following surfaces:
(a) the hemisphere \( x^2 + y^2 + z^2 = 1, \ x > 0 \) (which lies in front of the plane \( x = 0 \));
(b) the surface obtained by rotating the curve \( x^3 + 2y = 10, \ 1 \leq y \leq 5 \), about the \( x \)-axis.

Answers:
(a) either \( x = \sqrt{1 - y^2 - z^2} \), or
\[
\begin{align*}
x &= \cos \theta \sin \phi \\
y &= \sin \theta \sin \phi \\
z &= \cos \phi
\end{align*}
\]
with \( 0 \leq \phi \leq \pi \) and \( 0 \leq \theta \leq \pi \) (it is important to specify the domain, since otherwise it will describe the entire sphere).
(b)
\[
\begin{align*}
x &= x \\
y &= \frac{10 - x^3}{2} \cos \theta \\
z &= \frac{10 - x^3}{2} \sin \theta
\end{align*}
\]
with \( 0 \leq \theta \leq 2\pi \) and \( 0 \leq x \leq 2 \) (again, it is important to specify the domain, because otherwise the equations will describe a much longer curve).