1. (10 pts. each) Evaluate the following:
   (a) If \( z = e^y \sin x \) and
   \[
   \left. \frac{dx}{dt} \right|_{t=0} = 2, \quad x(0) = 0, \quad \left. \frac{dy}{dt} \right|_{t=0} = 1 = y(0),
   \]
   find \( \left. \frac{dz}{dt} \right|_{t=0} \).
(b) Calculate \( \text{div} \mathbf{F} \) for
\[
\mathbf{F} = \sin(x^2 + y) \mathbf{i} + ye^z \mathbf{j} + x \ln y \mathbf{k}
\]

(c) Compute the curl \( \mathbf{F} \) for \( \mathbf{F} := (xe^y - z) \mathbf{j} + xyz \mathbf{k} \).
(d) Let $E$ be the region described by $1 < x^2 + y^2 + z^2 < 4$. Evaluate the integral
\[
\iiint_{E} z \, dV
\]
by changing to spherical coordinates.

(e) Determine whether or not the vector field $\mathbf{F}(x, y, z) = xi + yj + zk$ is conservative. If it is conservative, find $f(x, y, z)$ in order that $\mathbf{F} = \nabla f$. 
2. Find all local maxima, minima, and saddle points for
\[ f(x, y) = y^3 - 6xy + x^3. \]
3. Find the volume of the solid bounded above by the surface $z = xy^2$ and below by the triangle in the $xy$-plane with vertices $(1, 0)$, $(0, 2)$, and $(2, 0)$.
4. Find the surface area of the part of the paraboloid $z = x^2 + y^2$ that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$. 
5. Suppose that $C$ consists of the line segment from $(0, 0)$ to $(1, 0)$, the line segment from $(1, 0)$ to $(1, 1)$, and the arc of the curve $x = y^2$ from $(1, 1)$ to $(0, 0)$. Use Green’s Theorem to evaluate

$$\oint_C (xe^{3x} - 4y^2) \, dx + (2xy + y \sin y^2) \, dy.$$
Extra Credit: Let the surface $S_1$ be the part of the sphere $x^2 + y^2 + z^2 = 5$ that lies inside the cylinder $x^2 + y^2 = 1$ and above the $xy$-plane. Let $S_2$ be the part of the plane $z = 2$ that lies inside the cylinder $x^2 + y^2 = 1$. If for some vector field $\mathbf{F}$,
\[
\iint_{S_1} \text{curl } \mathbf{F} \cdot d\mathbf{S} = 3,
\]
how does this fact relate to
\[
\iint_{S_2} \text{curl } \mathbf{F} \cdot d\mathbf{S} = ?
\]
(The amount of extra credit - if any - will depend upon how well you justify your answer.)
Closed Book. No calculators.
CIRCLE YOUR ANSWER. You must show your work and justify your answer to receive credit.

1. (a) (10 pts.) If \( \mathbf{u} = \frac{1}{2} \mathbf{i} + \frac{\sqrt{3}}{2} \mathbf{j} \) find the directional derivative \( D_{\mathbf{u}} \sin(xy) \).

(b) (5 pts.) If \( \mathbf{u} = a \mathbf{i} + b \mathbf{j} \), what are values of \( a \) and \( b \) (with \( a^2 + b^2 = 1 \)) that maximizes \( D_{\mathbf{u}} \sin(xy) \) at the point with \( x = 1, y = 0 \)?
2. (20 pts.) Let

\[ f(x, y) = xye^{-(3x+2y)}. \]

Find all critical points and classify as local maxima, local minima, or saddle points.
3. A helix is described by
\[ \mathbf{r}(t) := 3(\sin 2t)\mathbf{i} + 3(\cos 2t)\mathbf{j} - 4t\mathbf{k}. \]

(a) (6 pts.) Find the unit tangent vector \( \mathbf{T} \) at the point \((0, 3, 0)\) on the helix.

(b) (6 pts.) Find the (principal unit) normal vector \( \mathbf{N} \) at the point \((0, 3, 0)\) on the helix.

(c) (8 pts.) Find the plane containing the point \((0, 3, 0)\) and determined by the vectors \( \mathbf{T} \) and \( \mathbf{N} \) from parts (a) and (b), i.e., the osculating plane.
4. (15 pts.) Calculate the limit exists if it exists. If it does not exist, justify your answer.

(a) (Hint: Change to polar coordinates.)
\[
\lim_{(x,y) \to (0,0)} \frac{x^3 - xy^2}{x^2 + y^2}
\]

(b)
\[
\lim_{(x,y) \to (0,0)} \frac{x^2 - 2xy}{4x^2 + y^2}
\]

5. (15 pts.) Find \( \frac{\partial f}{\partial t} \) if
\[
f(x, y) = (\sin y) \ln(x^2 + 2), \quad \text{and} \quad x = 2 \cos(st), \quad y = 3s - 2t.
\]
6. An athlete puts a shot which leaves his hand 6 ft. above the ground at a 45 degree angle to the horizon and at a speed of $29\sqrt{2}$ ft./sec.

(a) (8 pts.) Find the position vector $\mathbf{r}(t)$, which describes the motion of the shot for any time $t$. \(^1\)

(b) (4 pts.) How many seconds later does the shot hit the ground?

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\(^1\)Assume that the only force acting on the body is gravity. Hint: The acceleration of gravity is $-32$ ft./sec\(^2\).
(c) (3 pts.) How far (horizontally) does the shot go?