1. The solid $E$ in the first octant of space lies above the surface $z = 2(x^2 + y^2)$ and below the sphere $x^2 + y^2 + z^2 = 9/4$. Calculate its volume.

10 points

2. Evaluate

$$\iiint_E x\,dV,$$

where $E$ lies between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 9$ in the first octant.

10 points
3. The lamina $D$ is defined by the inequalities $0 \leq x \leq 1$, $0 \leq y \leq 1$, and its mass density function is given by $\rho(x, y) = x^2 + y^2$. Compute its mass and the center of mass.

10 points

4. The solid $B$ lies inside the cylinder $x^2 + y^2 = 1$ and inside the ellipsoid $9x^2 + 9y^2 + z^2 = 36$. Calculate its volume.

10 points
5. Evaluate the integral by reversing the order of integration.

\[ \int_{0}^{9} \int_{y^{1/2}}^{3} e^{x^{3}} \, dx \, dy. \]

10 points

6. Calculate the integral

\[ \int_{1}^{4} \int_{1}^{2} \left( \frac{x^{2}}{y} + \frac{y^{2}}{x} \right) \, dy \, dx. \]

10 points
7. Find the minimum and maximum values of the function $f(x, y, z) = yz + xy$
subject to the constraints $xy = 3$ and $y^2 + z^2 = 9$.  

10 points

8. We know that $x$, $y$, and $z$ are positive numbers the sum of which is equal to 1. Maximize the value of $xy^2z^3$.  

10 points
9. Find the points on the ellipsoid \( x^2 + y^2 + 4z^2 = 1 \) where the normal line is parallel to the line connecting the points \( (3, -1, 0) \) and \( (5, 2\sqrt{2} - 1, 1) \).

10 points

10. Let \( z = y^2 \tan x, \ x = t^2 uv, \ y = u + tv^2 \). Find \( \partial z/\partial t, \partial z/\partial u, \) and \( \partial z/\partial v \) when \( t = 2, \ u = 1, \ v = 0 \).

10 points