Name:

Note: Grades will be posted outside the classroom door by Wednesday, May 5. If you want your grade posted, write a number by your name above (and remember the number). Your grade will be posted by that number. If there is no number, then the grade will not be posted - it’s your choice.

Closed Book. No calculators. Show your work.

1. (10 pts. each) Evaluate the following:
   (a) $\frac{\partial}{\partial z}[y \ln(x - e^{-3z})]$

(b) The directional derivative of $f(x, y) = \frac{\sin(x)}{\cos(y)}$ in the direction of $\mathbf{u} = 3\mathbf{i} - 4\mathbf{j}$ at the point $x = y = \pi/4$. 

Date: July 20, 2004.
(c) Compute the curl $\mathbf{F}$ for $\mathbf{F} := xy\mathbf{i} + yz\mathbf{j} + zx\mathbf{k}$.

(d) For $\mathbf{F}$ defined in (c), compute grad div $\mathbf{F}$. 
(e) Find \( \frac{dF}{dt} \big|_{t=0} \) if \( F(x, y) = e^{x^2-y} \) and \( x = 1 + \sin 3t, \ y = 2 \cos t. \)
2. (10 pts.) Find the volume of the solid in the first octant \((x \geq 0, y \geq 0, z \geq 0)\) bounded by the surfaces \(x + z = 3\) and \(y + z = 3\).
3. (10 pts.) Calculate the integral

\[ \iiint_E z^3 \sqrt{x^2 + y^2 + z^2} \, dV \]

where \( E \) is the solid hemisphere (1/2 of a ball) with center at the origin, radius 2, that lies above the \( xy \)-plane.
4. (a) (10 pts.) Setup and calculate directly the line integral

\[ \int_C x^2y \, dx + 3xy \, dy \]

where \( C \) consists of the line from \((-1, 0)\) to \((1, 0)\) and the parabola \(1 = x^2 + y\) from \((1, 0)\) to \((-1, 0)\).
(b) (10 pts.) Calculate the integral in part (a) using Green's Theorem.
5. (10 pts.) Use the Divergence Theorem to calculate the flux (i.e. the surface integral) of \( \mathbf{F} := x^2y \mathbf{i} - yz \mathbf{j} + z^2x \mathbf{k} \) across the surface of the box with vertices \((\pm 1, \pm 2, \pm 2)\).
1. Find the volume of the solid bounded by the surfaces \( z = x^2 + y^2 \) and \( z = \sqrt{x^2 + y^2} \).
2. Find the area of the surface \( z = y + \frac{2}{3}x^{3/2} \) that lies above the triangle in the plane \( z = 0 \) with vertices \((0, 0, 0), (2, 0, 0), \) and \((0, 1, 0)\).
3. Find the volume of the solid bounded by above by \(x^2 + y^2 + z^2 = 9\) and below by \(\phi = \pi/4\) (where \(\phi\) is a spherical coordinate, \(0 \leq \theta \leq 2\pi\), and \(0 \leq \rho < \infty\)).
4. A wire forms the triangle in the plane with vertices $(0,0)$, $(1,1)$, and $(1,0)$. If the density of the wire is given by $\rho(x,y) = xy$, what is the mass?

5. Use Green’s Theorem to evaluate the integral

$$\int_C (x^3 - y^3)dx + (x^3 + y^3)dy$$

where $C$ is the boundary of the region between $x^2+y^2 = 1$ and $x^2+y^2 = 4$. 
6. Evaluate the surface integral $\int \int_S \mathbf{F} \cdot d\mathbf{S}$ for the vector field

$$\mathbf{F}(x, y, z) = \frac{x^2 + y^2}{z + 1} \mathbf{i} - \frac{x^2 + y^2}{z + 1} \mathbf{j} + 2z \mathbf{k}$$

where $S$ is the part of the plane $x + y + z = 1$ in the first octant with downward orientation.
1. (a) (10 pts.) Find the rate of change of $f(x, y, z) = \cos(xy) + \sin(yz)$ at $(0, 0, \pi)$ in the direction of $2\mathbf{i} - 3\mathbf{j} + 2\sqrt{3}\mathbf{k}$.

(b) (5 pts.) In which direction is $f$ increasing most rapidly at $(0, 0, \pi)$? (Give the specific vector.)
2. (18 pts.) Let
\[ f(x, y) = x^3 - 6xy + y^3. \]
Identify all local maxima, minima, and saddle points. Justify your answer.

3. (17 pts.) Find the maximum value of \( f(x, y, z) = x + y \) on the surface of the ball of radius 2 and centered at the origin, i.e., \( x^2 + y^2 + z^2 = 4 \).
4. (The density of a thin plate represented by $x^2 + y^2 = 12$ is given by $\rho(x, y) = 60y$. If we cut from the plate a region $R$ represented by the portion of the plate above $y = x^2 / \sqrt{2}$, (a) (10 pts.) what is the mass of $R$?

(b) (5 pts.) Show how to calculate the center of mass of $R$. For this part you may write the integrals without evaluating them.
5. (18 pts.) Setup and evaluate an integral that will give the volume of the solid in the first octant bounded by $x + z = 2$ and $y + z = 2$.

6. (17 pts.) Use polar coordinates to find the volume of the solid bounded by $f(x, y) = 8 - y^2 - x^2$ and $g(x, y) = y^2 + x^2$.

Extra Credit (4pts. No partial credit.) Evaluate

$$\int_{0}^{\pi} \int_{0}^{\pi} \frac{\cos y}{y} dy dx.$$
Test 1  
MA227  
February 2, 2004  
Name:

Closed Book. No calculators. Show your work.

1. A particle is at the point \((x(t), y(t), z(t))\) at time \(t\) where \(x(t) = \cos(t-1)\), \(y(t) = \sin(t-1)\), and \(z(t) = 2t^{3/2}\).

   (a) (5pts.) What is the vector from the origin to the curve at any time \(t\)?

   (b) (10pts.) Give a vector that is tangent to the curve traced by the particle when it passes the point \((1, 0, 2)\) on the curve.

   (c) (10pts.) What is the length of the path traced by the particle as time passes from \(t = 0\) to \(t = 1\)?
2. (10pts.) If \( z = f(x, y) \), define 
\[
\frac{\partial z}{\partial y}
\].

If \( f(1, 2) = 3 \) and \( f(1, 2 \frac{1}{10}) = 5 \), what is an approximate value for 
\[
\frac{\partial z}{\partial y}(1, 2)
\]?

3. (10pts.) Find the limit, if it exists, or show that the limit does not exist:
\[
\lim_{(x, y) \to (0, 0)} \frac{xy}{x^2 + y^2}
\].
4. Calculate the following derivatives. Show your work.
   (a.) (15pts.) For $f(x, y) = \cos(y)/(x^2 + y^2)$, find $f_x$ and $f_y$.

   (b.) (15pts.) Use the chain rule to find $f_u$ if $f(x, y) = e^{x^2} y$ and $x = \ln(u^2 + v^2)$, $y = v/u$. 
(c.) (10pts.) Assuming that $z = f(x, y)$, find $\frac{\partial z}{\partial y}$, if $x^2 + y^3 + z^2 - 3xyz = 4$.

5. Let $z = f(x, y) = \sqrt{x^2 + y^2}$.
   (a.) (10pts.) Find the equation of the tangent plane at the point $(3, 4, 5)$ on the surface.

(b.) (5pts.) Use the tangent plane to approximate $f(3 \frac{1}{7}, 4)$. 