1. **Part I**

There are 6 problems in Part 1, each worth 4 points. Place your answer on the line to the right of the question. Only your answer on the answer line will be graded.

(1) Find \( \lim_{(x,y) \to (2,8)} \sqrt{x^2 + 2xy} \).

(2) Find the first order partial derivatives of \( f(x, y) = xe^y + 2y^2x^2 \).

(3) Find the linearization \( L(x, y) \) of \( F(x, y) = xy^2 - 2yx^3 - 7 \) at the point (1,2).

(4) Calculate the gradient of the function \( f(x, y, z) = xy + z^3 + ye^x \).

(5) The gradient of a function \( f \) is given by \( \nabla f = \left\langle x - 2y^2, 4 - 2x \right\rangle \). Find all the critical points of \( f \).

(6) Find the directional derivative in direction \( v = (1, 2) \) of the function \( f(x, y) = x^3 - xy^2 \) at point (1,1).
2. Part II

There are 3 problems in Part 2, each worth 12 points. On Part 2 problems partial credit is awarded where appropriate. Your solution must include enough detail to justify any conclusions you reach in answering the question.

(1) Consider the function \( f(x, y) = x^2 y^4 + 2x - y^3 \).

(a) Compute \( f(1, -2) \).

(b) Find the equation of the tangent plane to the surface \( f(x, y) = z \) at the point \( P(1, -2) \). (Hint: An easy way to get the tangent plane is calculating the linearization at \( P \) and set \( z = L(x, y) \))
(2) Use Lagrange multipliers to find the minimal and maximal values of $f(x, y) = x + 2y$ on the ellipse $\frac{1}{5}x^2 + y^2 = 1$. Where do they occur?
(3) (a) Identify which of the points \( P(1, -1), Q(2, -4), R(-3, -9), S(2, -1) \) are critical points of \( f = x^2 + 14y^2 - 24y + 2xy^2 + 11 \).
(b) Classify the critical points of \( f \) found in (a): local min., local max., saddle point.