TEST 3

10 questions, 10 points each. SHOW ALL YOUR WORK!

Question 1

Find $\int \int_D x \, dxdy$, where $D$ is bounded by $y = x^4$ and $y = 8x$. 
Question 2

Find the volume under the surface $z = x^2y$ and above the triangle in the $xy$ plane with vertices $(0, 0), (1, 0), (1, 1)$. 
Question 3

Sketch the region of integration and change the order of integration:

\[ \int_0^1 \int_{x^4}^1 f(x, y) \, dy \, dx. \]
Question 4

Use polar coordinates to find the volume under the plane $z = 2x + y + 8$ and above the half-disk $x^2 + y^2 \leq 9$, $x \geq 0$ in $xy$ plane.
Question 5

Find the mass of the lamina that occupies the region:

\[ D = \{(x, y) | x^2 + y^2 \leq 1, \ y \geq 0\} \]

and has the density function given by \( \rho(x, y) = x^2 + y^2 \).
Question 6

Evaluate the iterated integral \( \int_{0}^{1} dx \int_{0}^{x} dy \int_{0}^{x+y} dz. \)
Question 7

Express the integral $\int \int \int_{E} f(x, y, z) dV$ as an iterated integral, where $E$ is the solid above the region $D = \{(x, y) : y^4 \leq x \leq 1\}$ in $xy$ plane and below the plane $z = x + y + 2$. 
Question 8

Find \( \int \int_D (x + 2y) \, dxdy \), where \( D \) is bounded by \( x + 2y = 0, x + 2y = 4, x - y = 0, x - y = 2 \).
Use change of variables \( u = x + 2y, v = x - y \).
Question 9

For the integral $\int \int_D f(x, y) \, dx \, dy$ consider the change of variables $u = xy$, $v = \frac{x}{y}$. Find inverse change of variables $x = x(u, v)$, $y = y(u, v)$ and calculate corresponding Jacobian. You DO NOT need to substitute it into integral.
Question 10

Using TWO DIFFERENT ITERATED INTEGRALS find area of the domain $D$ bounded by $y = x^2$, $y = 9$. Be sure that you get the same answer in both cases.