TEST II

10 questions, 10 points each.
SHOW ALL YOUR WORK! CIRCLE YOUR ANSWER!

Question 1

Find the gradient of the function \( f(x, y) = x^2e^{-xy} \) at the point \((1, 0)\).

Question 2

Find the directional derivative of the function \( f(x, y, z) = xz^2 + yz \) in the direction of the vector \( \vec{v} = \vec{i} + 2\vec{j} - 2\vec{k} \) at the point \((2, 1, -1)\).
Question 3

Find local maximum, minimum and saddle points (if any) of the function

\[ f(x, y) = x^2 - 6xy + 4y^2 - 5y + 1. \]
Question 4

Let \( z = xy^3 - \frac{1}{y} \). Find equation of the tangent plane at point \((-1, 1)\).

Question 5

Find linear approximation for the function

\[
f(x, y) = x^2 - xy + y^3 x
\]

near point \((2, 1)\).
Question 6

Let $f(x, y) = xy + \frac{x^2}{y}$ and $x = st, \quad y = s^2 + t$. Find partial derivatives $\frac{\partial f}{\partial s}$ and $\frac{\partial f}{\partial t}$.

Question 7

Let $f(x, y) = y\sin(x) + x^2y$. Find all second partial derivatives: $f''_{xx}, \ f''_{xy}, \ f''_{yy}$. 
Question 8

Find equation of the tangent plane to the surface $x^3 - y^3 + z^2 = -8$ at the point $(-1, 2, 1)$.

Question 9

Find the maximum rate of change of $f(x, y) = y\sqrt{x} - \frac{z}{y}$ at the point $(1, -1)$. In which direction does it occur?
Question 10

Find the absolute maximum and absolute minimum of the function $f(x, y) = x^2 + 2y^2 - 2x + 1$ on the region $-2 \leq x \leq 0$, $-1 \leq y \leq 1$. Be sure to provide coordinates of the points and the values of absolute maximum and minimum.