MA-227/6D: Calculus III
Test #1, February 7, 2011

Time available: 110 min

Your name (print):

Your signature:

Please always explain your answer, at least by including your calculations. You should work on this sheet. A right answer without calculation brings you no credit!

1. At what point(s) does the curve
   \[ \vec{r}(t) = \langle t - 2, 11t^2 - 11t + 14, 3t \rangle \]
   intersect the paraboloid \( y = 3x^2 + z^2 \)?
   
   10 points

2. Find a parametric equation for the tangent line to the previous curve at the point \( P(0, 36, 6) \).
   
   10 points
3. Find the curvature of the curve \( \vec{r}(t) = \cos t \vec{i} + \cos t \vec{j} - 3 \sin t \vec{k} \) at the point \( P(1,1,0) \).

    20 points

4. Find the vectors \( T, N, \) and \( B \) for the curve of problem 3 at the given point.

    20 points
5. Find the tangential and normal components of the acceleration vector for the curve \( \vec{r}(t) = t\vec{i} + 2t\vec{j} + t^2\vec{k} \) at the generic point \( \vec{r}(t) \). 

10 points

6. The motion \( \vec{r}(t) \) takes place for positive time (always \( t > 0 \)), \( \vec{a}(t) = 6t\vec{i} + \frac{1}{t^2}\vec{j} + 6t\vec{k} \), \( \vec{v}(1) = 3\vec{i} - \vec{j} + 3\vec{k}, \vec{r}(1) = \vec{i} + \vec{k} \). Compute \( \vec{r}(t) \).

20 points
7. Find the vectors $\vec{T}(t)$, $\vec{N}(t)$, and $\vec{B}(t)$ for the curve

$$\vec{r}(t) = \langle 3t, 4\cos t, 4\sin t \rangle.$$  

10 points

8. Find the velocity, acceleration, and speed of a particle with the position function

$$\vec{r}(t) = \langle t, t\sin t, -t\cos t \rangle.$$  

10 points
9. Find the curvature of the space curve

\[ \vec{r}(t) = t\vec{i} + tj - t^2\vec{k} \]

at any point.

10. Let \( r(t) = \langle t, 3t, t^2 \rangle \). Find the tangential and normal components of the acceleration, i.e. find \( a_T \) and \( a_N \).

10 points