There are 10 questions, each worth 11 points; 100 (or more) points is equivalent to 100% for the exam. Partial credit is awarded where appropriate. Show all working; your solution must include enough detail to justify any conclusions you reach in answering the question.

1. (a) Find the equation of the plane containing the points $(1, -1, 2), (1, 1, -1)$ and $(-1, 0, 1)$.
   
   (b) Let $\mathbf{r}(t) = (t^3, 1, e^{t^2-1})$. Find the unit tangent vector at the point on the curve corresponding to $t = 1$. 
2. (a) Find the directional derivative of the function \( f(x, y, z) = xz - yz^2 \) in the direction of the vector \( \vec{v} = \vec{i} + \vec{j} - 2\vec{k} = \langle 1, 1, -2 \rangle \) at the point \((-1, 0, 1)\).

(b) Find the maximum rate of change of \( f(x, y) = xy^3 - 4\sqrt{x} \) at the point \((1, -1)\). In which direction does it occur?
3. (a) Let \( z = y^2 - x^3y \). Find the equation of the tangent plane at the point \((1, -2)\).
(b) Find equation of the tangent plane to the surface \( x^2 + 2y - xz^2 = 4 \) at the point \((1, 2, 1)\).
4. Find the local maximum, minimum and saddle points (if any) of the function
\[ f(x, y) = x^2 - 4xy + y^2 - 2y + 2. \]
5. (a) Find the linear approximation for the function

\[ f(x, y) = ye^{x-2} - x^2 y^3 \]

near the point (2, 1).

(b) Let \( f(x, y) = x^2 y - e^y \) and \( x = s - t, \quad y = s^2 t \). Find the partial derivatives \( \partial f/\partial s \) and \( \partial f/\partial t \). You don’t need to simplify your answer!
6. Find the absolute maximum and absolute minimum points of the function

\[ f(x, y) = x^2 - y^2 + y \]

on the region \( 0 \leq x \leq 1, \ -1 \leq y \leq 1 \). Be sure to provide the coordinates of the points and the values of absolute maximum and minimum.
7. Evaluate the integral
\[ \iint_D (x - y)^3 \sin(x + 2y) \, dA \]
where \( D \) is the parallelogram enclosed by the lines \( x - y = 0, \ x - y = 1, \ x + 2y = 0, \) and \( x + 2y = \frac{\pi}{4} \). Use the change of the variables \( u = x - y, \ v = x + 2y \).
8. (a) Switch the order of integration in the iterated integral

\[ \int_{x^2}^{1} \left[ \int_{x}^{1} f(x, y) \, dy \right] \, dx. \]

(b) Using a double integral, find the area of the triangle with vertices \((0, 0), (1, 1), (0, 2)\).
9. (a) Change \((1, -\sqrt{3}, -2\sqrt{3})\) from rectangular into spherical coordinates.
(b) Using spherical coordinates evaluate
\[
\iiint_{E} (x^2 + y^2 + z^2)^2 \, dV,
\]
where \(E\) is the half-ball \(x^2 + y^2 + z^2 \leq 1, \ x \leq 0\).
10. Use polar coordinates to find the mass of the lamina that lies within the region \( x^2 + y^2 \leq 4, \ 0 \leq x \leq y \), if the material in the lamina has density (mass per unit volume) given by \( \rho(x,y) = x^2 + y^2 \).