MA-227/6D: Calculus III
Test #2, November 18, 2013

Time available: 110 min.
Each problem is 15 points

Your name (print):

Your signature:

Please always explain your answer, at least by including your calculations. You should work on this sheet. A right answer without calculation brings you no credit!

1. Calculate
\[ \int \int_R x^3 ye^{x^4} y \, dA, \]
\[ R = [0, 1] \times [0, 2]. \]

2. Find the volume of the solid bounded by the cylinder \( x^2 + y^2 = 1 \) and the planes \( z = 0, z = x + 5 \).
3. Evaluate the integral by converting to polar coordinates.

\[
\int_{-1}^{1} \int_{0}^{\sqrt{1-x^2}} \sin(x^2 + y^2) \, dy \, dx.
\]

4. Let the lamina \( D \) be bounded by the curves \( y = e^{-x}, \ y = 0, \ x = 0, \ \text{and} \ x = 1 \) with mass density function \( \rho(x, y) = 3 \). Find the moment of inertia \( I_x \) and the total mass \( m \).
5. Find the volume of the solid enclosed by the paraboloid \( z = 25 - x^2 - y^2 \) and the plane \( z = 0 \).

6. Evaluate the integral by switching to cylindrical coordinates.

\[
\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{0}^{\sqrt{4-x^2}} 2(x^2 + y^2) \, dz \, dy \, dx.
\]
7. Calculate
\[ \iiint_E zdV, \]
where \( E \) lies between the spheres \( x^2 + y^2 + z^2 = 1 \) and \( x^2 + y^2 + z^2 = 9 \) in the first octant.

8. Find \( \iint_D xdA \), where
\[ D = \{ (x, y) | 0 \leq x \leq 1, \ x^6 \leq y \leq x^4 \}. \]