Problem 1 (20 points)

Solve the initial value problem

\[ y'' + 2y' + 5y = 0, \quad y(0) = 1, \quad y'(0) = 6 \]
Problem 2 (20 points)

Solve the initial value problem

\[ y'' - 3y' - 4y = 5, \quad y(0) = -\frac{1}{4}, \quad y'(0) = 0 \]
Problem 3 (20 points)

Use Variation of Parameters to find the general solution of

\[ y'' - 2y' + y = x^2 e^x. \]

(Recall the formulas

\[ u_1' = \begin{vmatrix} 0 & y_2 \\ f & y_2' \\ y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}, \quad u_2' = \begin{vmatrix} y_1 & 0 \\ y_1' & f \\ y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}, \]

used in variation of parameters.)
Problem 4 (30 points)

A mass of 2 kilograms stretches an undamped spring by 2 meters and 45 centimeters.  
(a) Find the equation of motion if the mass is released 50 cm below the equilibrium position 
at a downward velocity of 1 m/s. Assume here that the positive $x$-direction is oriented downwards.  
(b) Write the equation of motion in the form $x(t) = A \sin(\omega t + \phi)$ and determine $A$ and $\phi$.  
(c) Find the first positive time at which the mass passes through the equilibrium position.
Problem 5 (10 points)

Suppose that a damping force is added to the spring/mass system in Problem 4 which is numerically equal to $b$ times the instantaneous velocity, where $b$ is a constant. How does $b$ have to be chosen for the system to become critically damped?