TEST I

Problem 1 (16P)

Determine the order of the following ODEs. Also, state if they are linear or non-linear.

(a) $y' + \sin(y) = x$
(b) $y^{(6)} - y^{(3)} = \cos(xy)$
(c) $yy'' = e^x$
(d) $\frac{y - \sin(x)}{x^2y'} = e^x$

Problem 2 (12P)

(a) Which of the following functions are solutions of $x^4y' + 2xy^2 = 4x^5$? (8P)

$y_1 = -x^2, \quad y_2 = x, \quad y_3 = x^2, \quad y_4 = -2x^2.$
(b) Which of the functions from part (a) solve the initial value problem \( x^4y' + 2xy^2 = 4x^5, \)
\( y(0) = 0 \)? (4P)

(c)* (Bonus) Does your answer to part (b) agree with the content of the Existence and
Uniqueness Theorem for first order ODEs? If yes, why? If no, why not? (5P*)

Problem 3 (12P)

(a) In the 3 \( \times \) 3-grid of points \( x = 0, 1, 2 \) and \( y = 0, 1, 2 \) provided in the figure below draw a
direction field for \( y' = x^2(y - 2) \). (8P)

(b) Without solving the DE, use the direction field to read off the solution of the IVP
\( y' = x^2(y - 2), y(1) = 2 \). (4P)
Problem 4 (12P)

Solve the IVP
\[ y' = x^2(y - 2), \quad y(1) = 1. \]

Problem 5 (14P)

Solve the IVP
\[ \frac{y'}{x} + y = 1, \quad y(0) = 3 \]
Problem 6 (12P)

Solve the IVP

\[ y' - y^2 \sin(x) = 0, \quad y\left(\frac{\pi}{3}\right) = 2 \]
Problem 7 (12P)

The mass of a radioactive material is given by \( m(t) \), where the time \( t \) is measured in years and the mass in grams. An initial mass of \( m(0) = 100 \) grams decays at a constant rate \( k = m'(t)/m(t) \). After 1 year 80 grams of the material are left.

(Note: Your answers will contain natural logarithms which do not need to be evaluated.)

(a) Find the decay rate \( k \) by solving the differential equation for \( m(t) \). (8P)

(b) Find an expression for the time \( t_h \) at which only 50 grams of the material are left (\( t_h \) is the so-called half-life of the material). (4P)
Problem 8 (10P)

In the electrical circuit below one has \( R = 100 \) ohms, \( C = 0.01 \) farads and a constant electromotive force of \( E(t) = 100 \) volts.

(a) Write down the DE for the charge \( q(t) \) and solve it to find a closed form solution for \( q(t) \) in Coulombs given that \( q(0) = 0 \). (6P)

(b) Find a formula for the current \( i(t) \) in amperes. (4P)
SCRATCH PAPER

(Scratch paper will not be graded!)
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