Problem 1 (9+9 points)

(a) Let $f(x, y) = \frac{2x}{x-y}$. Find $f_{xx} + f_{yy}$.

(b) For the function $g(x, y) = x \ln(x^2 + y)$ find $g_x$, $g_y$ and $g_{xy}$. 
Problem 2 (9+9 points)

(a) For the function $h(x, y) = e^{(x^4 + y^2)}$ find its direction and rate of steepest descent at the point $P(1, 2)$.

(b) Find the directional derivative of $h(x, y)$ at $P(1, 2)$ in the direction of the vector from $P(1, 2)$ to $Q(4, -2)$. 

Problem 3 (12+6 points)

(a) Find an equation for the tangent plane to the graph of \( z = xy + \cos(x - y) + 1 \) at the point \( \left( \frac{\pi}{2}, 0, 1 \right) \).

(b) Also, find parametric equations for the normal line to the graph of \( z = xy + \cos(x - y) + 1 \) at \( \left( \frac{\pi}{2}, 0, 1 \right) \).
Problem 4 (12 points)

Evaluate \( \int_C x^3 y \, ds \), where \( C \) is a quarter of a unit circle parametrized by \( x = \cos(t) \), and \( y = \sin(t) \), \( 0 \leq t \leq \frac{\pi}{2} \).

Problem 5 (12 points)

Find the work done by the force field

\[
F(x, y) = \frac{\ln(x)}{x} \mathbf{i} - x\sqrt{y} \mathbf{j}
\]

along the curve \( C \) given by the graph of \( y = x^2 \), \( 1 \leq x \leq 2 \).
Problem 6 (5+5 points)

Determine for each of the following force fields if it is conservative.

(a) \( F(x, y) = y^2 \sin(x)i + \frac{y^3}{3} \cos(x)j \)

(b) \( F(x, y) = (e^x + xy^2)i + (x^2y + e^y)j \)

Problem 7 (12 points)

For the conservative force field \( F(x, y) \) from Problem 6 find a potential function \( \phi(x, y) \) and calculate the work done by the force field along the curve traced by the vector function \( \mathbf{r}(t) = (1 + t^2)i + t\sqrt{7}j, \ 0 \leq t \leq 1 \).