No calculators are allowed!

11 questions, 10 points each. SHOW ALL YOUR WORK!

Question 1

a) Evaluate the integral \( \int_{-1}^{6} (x + 2)^{1/3} \, dx \).

b) Calculate the derivative of \( y = \tan^{-1}(\ln(x)) \).
Question 2

Evaluate the integral $\int xe^{2x} \, dx$.

Question 3

Evaluate the integral $\int (\sin(x))^2 (\cos(x))^3 \, dx$. 
Question 4

The acceleration of the particle is given by $a = 2t$, find velocity and position of the particle if velocity at time $t = 1$ is $v(1) = 1$ and position at time $t = 1$ is $r(1) = 0$.

Question 5

Find the limit

$$\lim_{{x \to 0}} \frac{e^{-x} - 1}{\sin(x)}$$
Question 6

Write the rational fraction

\[ f(x) = \frac{x^2 + 1}{x(x - 1)(x - 2)} \]

as a sum of partial fractions. Use this representation to calculate the integral \( \int f(x) \, dx \) and write the answer as a single logarithm.
Question 7

Evaluate the integral

\[ \int x^2 \ln(x^3) \, dx \]
Question 8

Evaluate the limit

$$\lim_{x \to 0} (1 + x)^{\frac{1}{\sin(x)}}$$
Question 9

Determine whether the improper integral converges. Give your reasons! You DO NOT need to calculate the integral.

a) \[ \int_{1}^{\infty} \frac{x^{10}e^{x}}{x^{2} + e^{2x}} \, dx \]

b) \[ \int_{1}^{\infty} \frac{x}{x^{2} + 1} \, dx \]
Question 10

Evaluate the improper integral
\[ \int_1^\infty \frac{\ln(x)}{x^2} \, dx \]

Question 11

Evaluate the improper integral
\[ \int_0^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}} \, dx \]