TEST II

Closed book - Calculators and One Index Card are allowed!

PART I

Part I consists of 5 questions. Clearly write your answer (only) in the space provided after each question. Show your work to justify your answers. Very limited partial credit or none at all for this part of the test!

Each question is worth 8 points.

Question 1

Determine whether the improper integral \( \int_{e}^{\infty} \frac{1}{x(\ln(x))^{0.2}} \, dx \) is convergent or divergent. Find its numerical value if it converges!

Answer: 

\( \text{Answer: } \)
Question 2

Sketch the region enclosed by the curves $y = |2x|$ and $y = x^2 - 3$, and then find its area.

Answer: ..................

Question 3

Find the volume of the solid obtained by rotating about the $x$-axis the region bounded by the curve $y = 2 - 2x^2$ and the line $y = 0$.

Answer: ..................
Question 4

Sketch the region described below, and use the method of cylindrical shells to write out an integral-formula for the volume of the solid generated by rotating the region bounded by the curve $y = 3x - x^2$ and the line $y = x$ about the $y$-axis. (Do NOT compute the integral you obtain!)

Answer: ..................

Question 5

A variable force $f(x) = 2x^{-2}$ N moves an object along a straight line when it is $x$ meters from the origin. Calculate the work done in moving the object from $x = 1$ m to $x = 10$ m.

Answer: ..................
PART II

Each problem is worth 15 points.

Part II consists of 4 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit - no credit for unsubstantiated answers!

Problem 1

Use the graph below to set up and write out the mid-point rule approximation $M_6$ of the integral $\int_0^6 v(t) \, dt$ with 6 sub-rectangles. (Do NOT add up the terms!)
**Problem 2**

Determine how large the number $a$ has to be so that the area under the graph of the function $f(x) = \frac{1}{1 + x^2}$ is less than or equal to $\pi/6$ for $x \geq a$; that is, determine how large the number $a$ has to be so that

$$\int_a^\infty \frac{1}{1 + x^2} \, dx \leq \frac{\pi}{6}.$$  

(Give the exact answer! NO approximation.)
Problem 3

Find the volume of the solid obtained by rotating the region bounded by the curves $y = x^3$ and $y = \sqrt{x}$ about the line $x = -1$. 
Problem 4

A cable that weighs 4 lb/ft is used to lift 700 lb of coal up a mine-shaft 650 ft deep.

(a) Find the work needed to lift the cable (only) to the top.

(b) Find the work needed to lift the coal (only) to the top.

(c) Find the total work required to lift both the cable and the coal to the top.
SCRATCH PAPER

(Scratch paper will not be graded!)
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