Part I consists of 8 questions. Clearly write your answer (only) in the space provided after each question.
Show your work to justify your answers. Very limited partial credit or none at all for this part of the test!

Each question is worth 6 points.

Question 1

Evaluate the definite integral $\int_{0}^{3} \sqrt{1 + x} \, dx$. (Give the exact answer. No approximation.)

Answer: .....................

Question 2

Evaluate the definite integral $\int_{1}^{e} \frac{2}{x} \, dx$. (Your answer must be a real number!)

Answer: .....................
Question 3

Evaluate the indefinite integral $\int x \cos(x) \, dx$.

Answer: ................

Question 4

The velocity of a particle is given by $v(t) = 3t^2 + 2t$ and its position at time $t = 0$ is 4. Find its position at time $t = 1$.

Answer: ................

Question 5

Evaluate the indefinite integral $\int \sin^3(x) \cos(x) \, dx$.

Answer: ................
Question 6

Evaluate the definite integral \( \int_{0}^{1} \frac{x}{x+1} \, dx \). (Give the exact answer. **No approximation!**) 

Answer: ....................

Question 7

Find the derivative of the function \( y = \tan^{-1}(x^3) \). Also state the differentiation rule that you use!

Answer: ....................

Question 8

Simplify the expression \( \tan(\sin^{-1}(x)) \); that is, write the given expression in terms of the independent variable \( x \)!

Answer: .....................
PART II

Each problem is worth 13 points.

Part II consists of 4 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit - no credit for unsubstantiated answers!

Problem 1

Determine whether or not each of the following limits exist. Find the limit if it does exist or justify why the limit does not exist. Simplify where appropriate! (Always show your work!)

\[ \lim_{x \to 0} \frac{9e^x - 9}{\sqrt{3x}}. \]

\[ \lim_{x \to \infty} x - x^2 \]

\[ \lim_{x \to \infty} x^2 e^{-x} \]
Problem 2

Evaluate the indefinite integral

\[ \int 2x \ln(x^2) \, dx. \]

(Show your work!)
**Problem 3**

Evaluate the indefinite integral

\[ \int \sin^2(x) \cos^3(x) \, dx. \]

(Show your work!)
Problem 4

(a) Write the rational function \( \frac{3x^2 - 1}{x(x + 1)(x - 1)} \) as a sum of partial fractions; that is, find the numerical values of the constants \( A, B \) and \( C \) such that

\[
\frac{3x^2 - 1}{x(x + 1)(x - 1)} = \frac{A}{x} + \frac{B}{x + 1} + \frac{C}{x - 1}.
\]

(b) Use the above to evaluate the following indefinite integral. (Do not simplify your answer!)

\[
\int \frac{3x^2 - 1}{x(x + 1)(x - 1)} \, dx.
\]

(c) Write the logarithms included in your answer in part (b) as a single logarithm.
SCRATCH PAPER

(Scratch paper will not be graded!)
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