

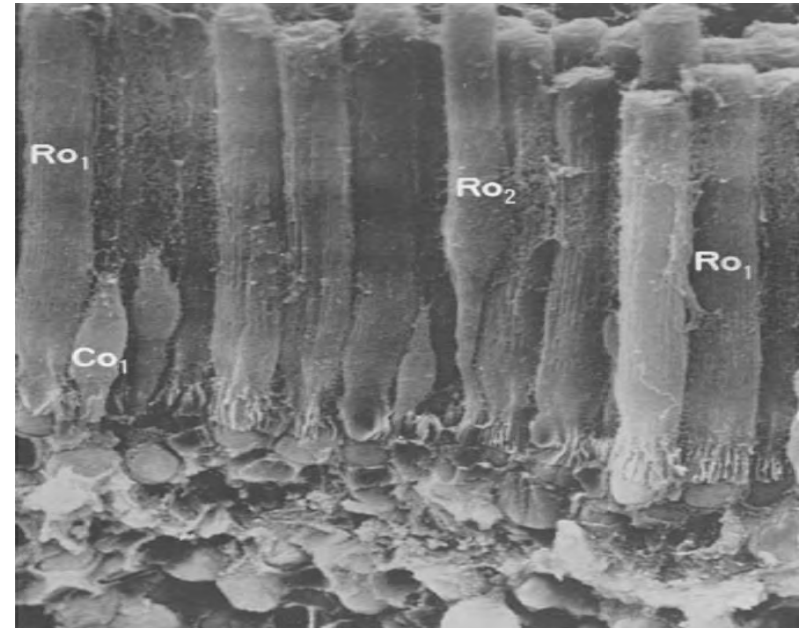
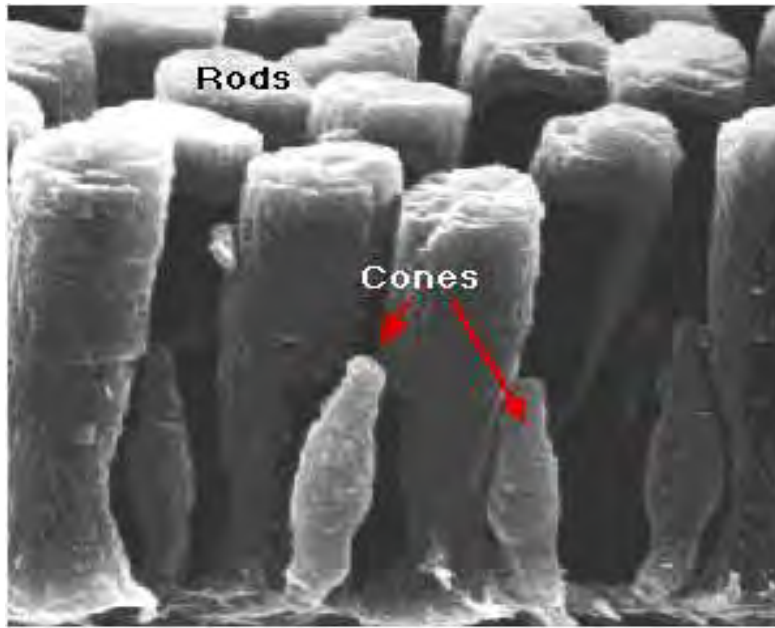
# Visual Transduction: A Paradigm for Signaling

E. DiBenedetto

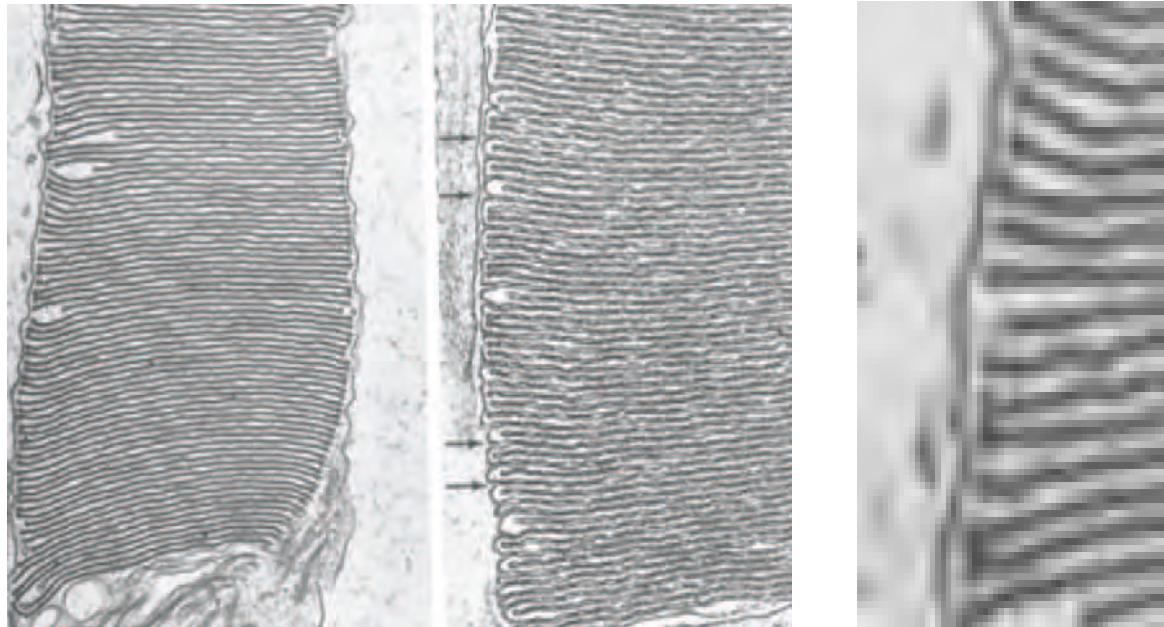
`em.diben@vanderbilt.edu`

Department of Mathematics, Vanderbilt University  
Nashville, TN 37240, USA

# Micrograph of Salamander and Frog Retina



## Micrograph of the Salamander Rod Outer Segment

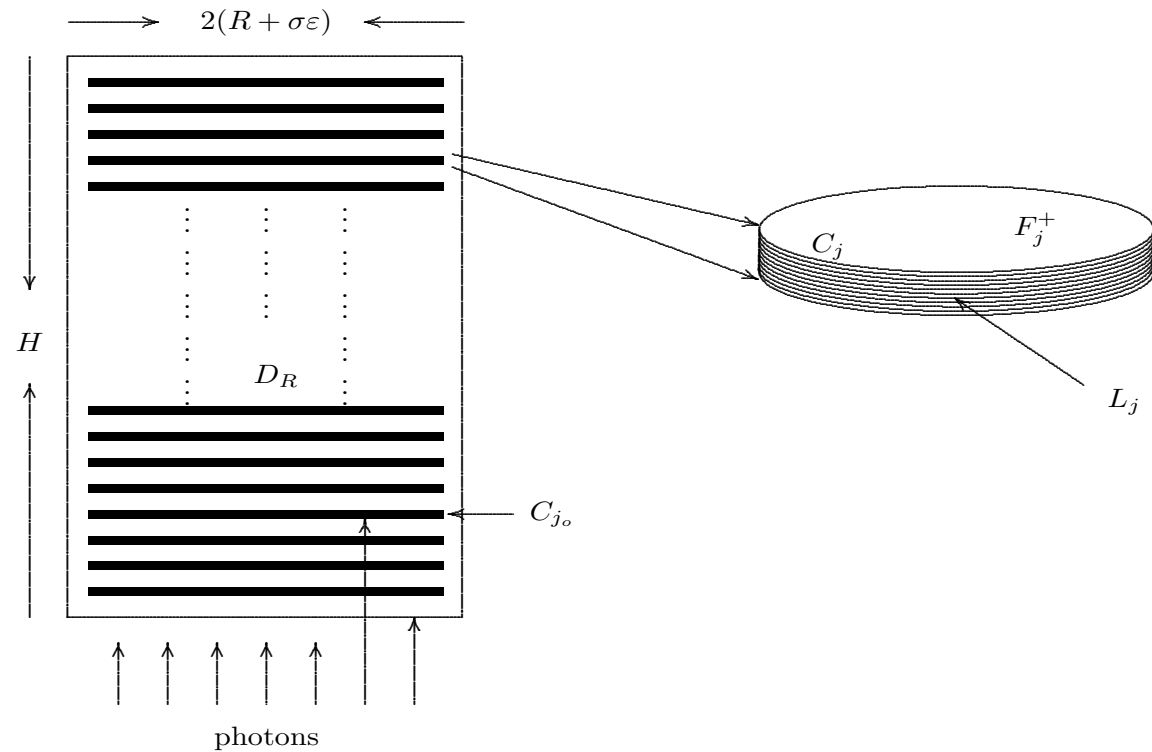


Thickness  $\sigma\epsilon_o$  of outer shell comparable to distance  $\nu\epsilon_o$  between any two discs. For the Salamander  $\epsilon_o \approx 14\text{ nm}$  and  $\nu \approx \sigma \approx 1$ . Here  $\epsilon_o$  is the thickness of the “layers/discs”.

# Geometry of the Salamander ROS

$$H \approx 22\mu\text{m} \quad R \approx 5.5\mu\text{m} \quad \varepsilon_o \approx 14\text{ nm}$$

$$\nu \approx 1 \quad \sigma \approx 1 \quad n \approx 800$$

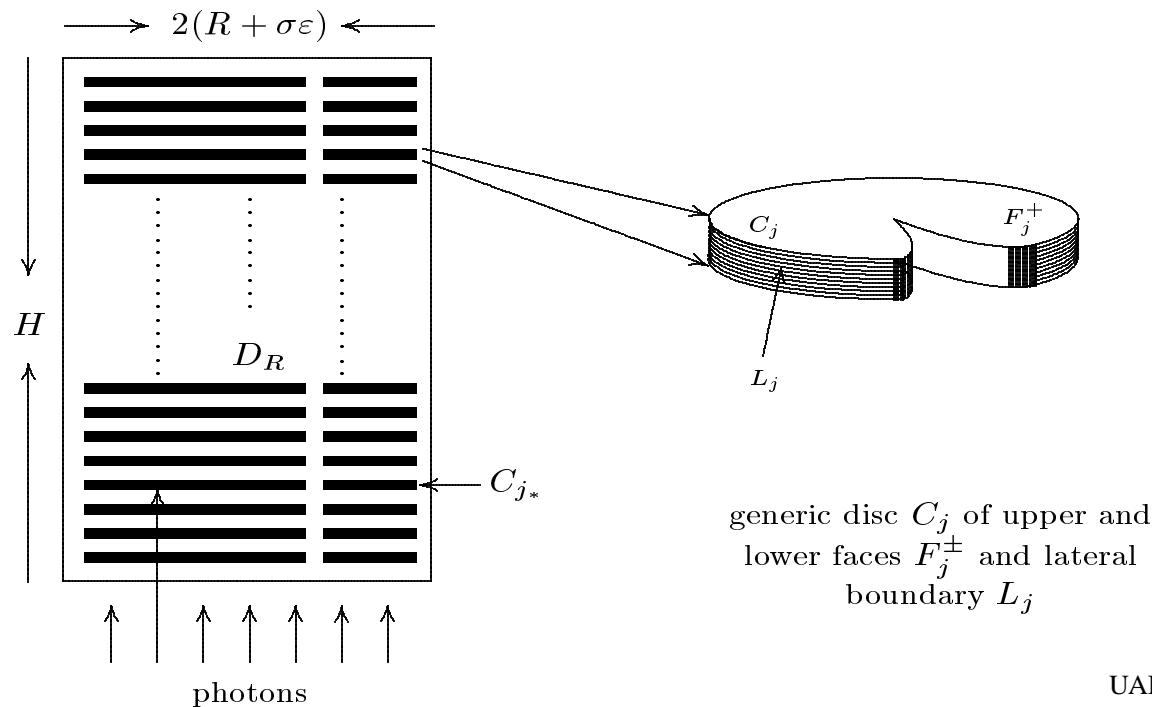


# Geometry of the ROS with Incisures

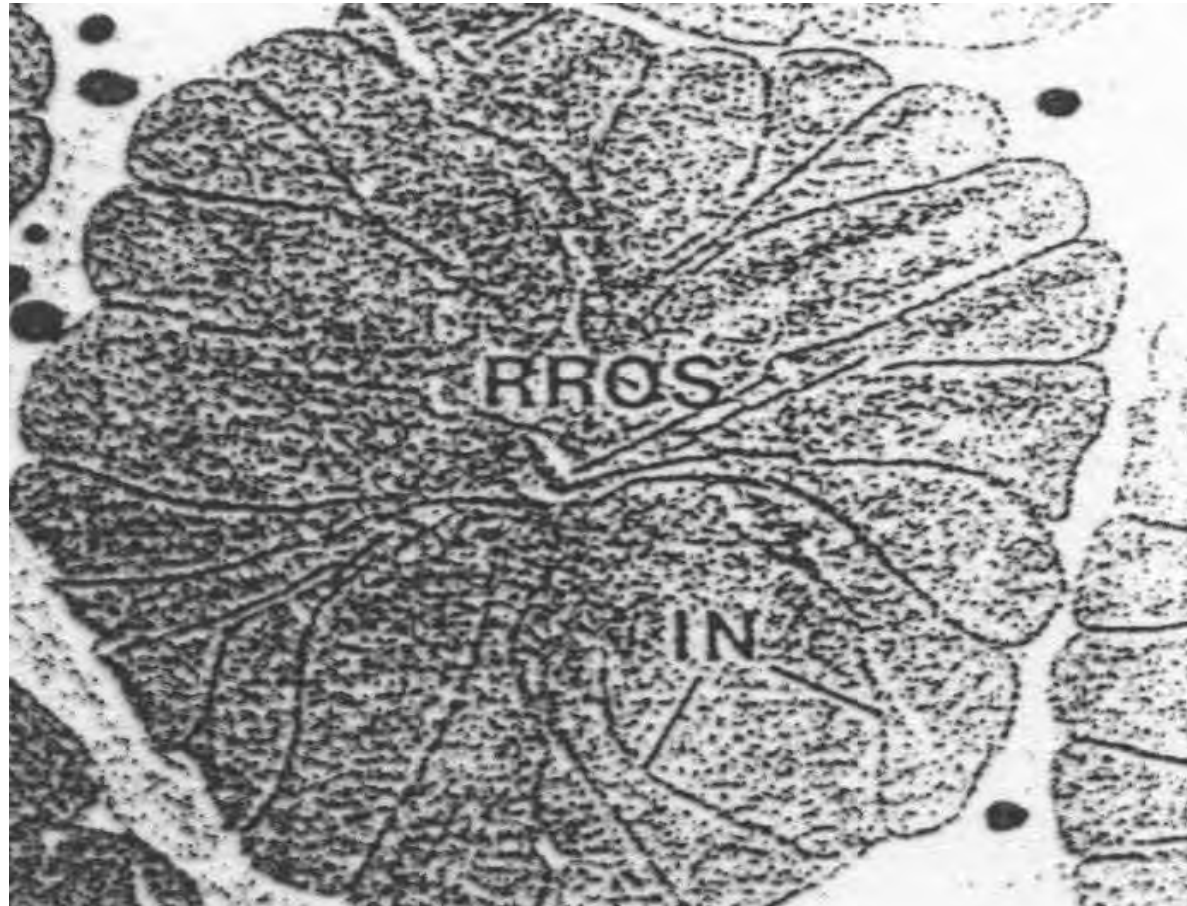
$S_\varepsilon = \{D_{R+\sigma\varepsilon} - D_R\} \times (0, H)$  Outer Shell  $\varepsilon \leq \varepsilon_o$

$I_j = \{\text{interdiscal spaces}\}$  thickness  $\approx \nu\varepsilon$   $\nu \approx 1$

$\mathcal{B}_\varepsilon = \mathcal{V}_\varepsilon \times (0, H)$  blade-like incisures  $n \approx 800$



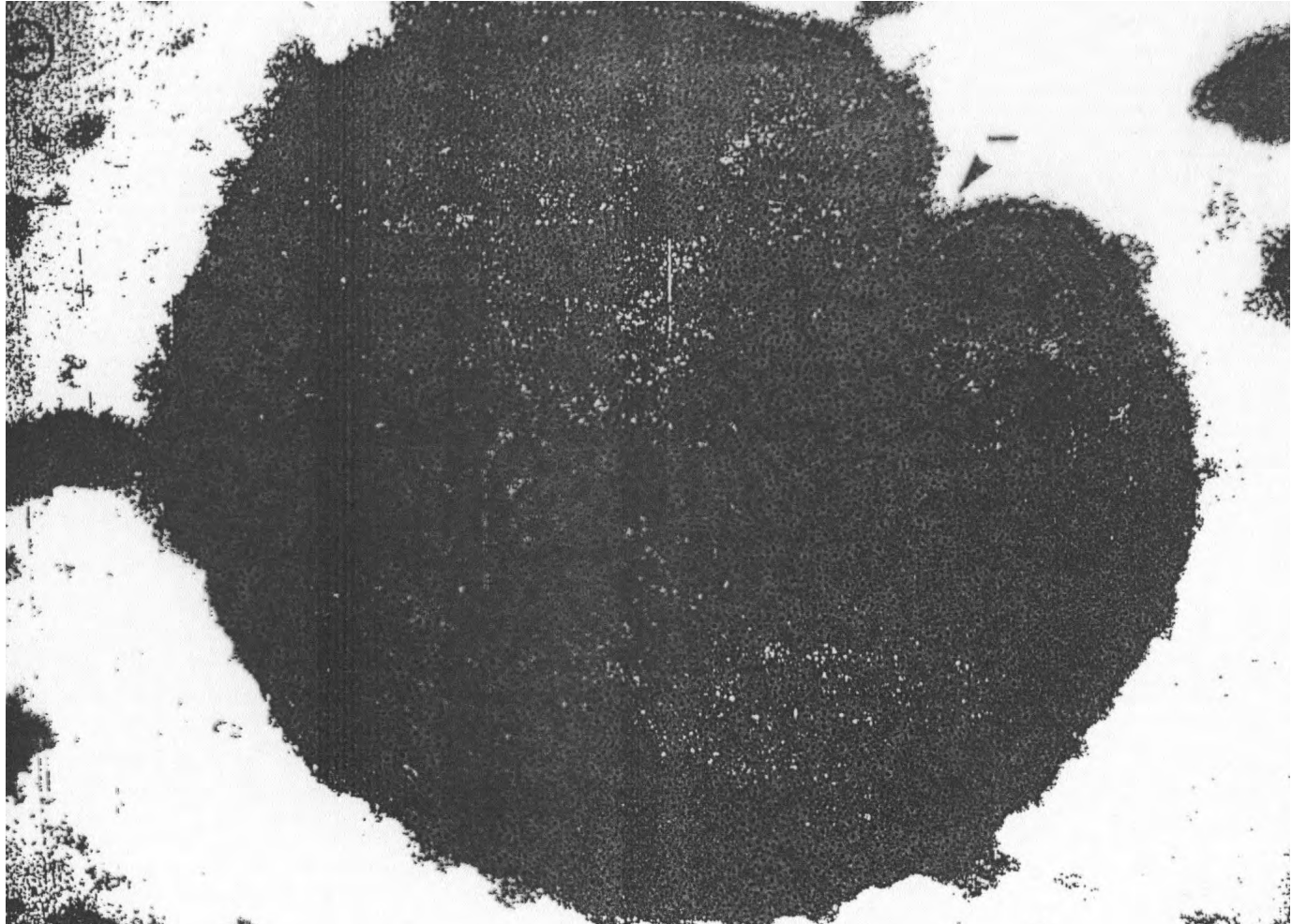
## Incisures in Frog ROS



Vision Res. Vol. 22, 1982, pp. 1417–1428



## Incisures in Rat ROS



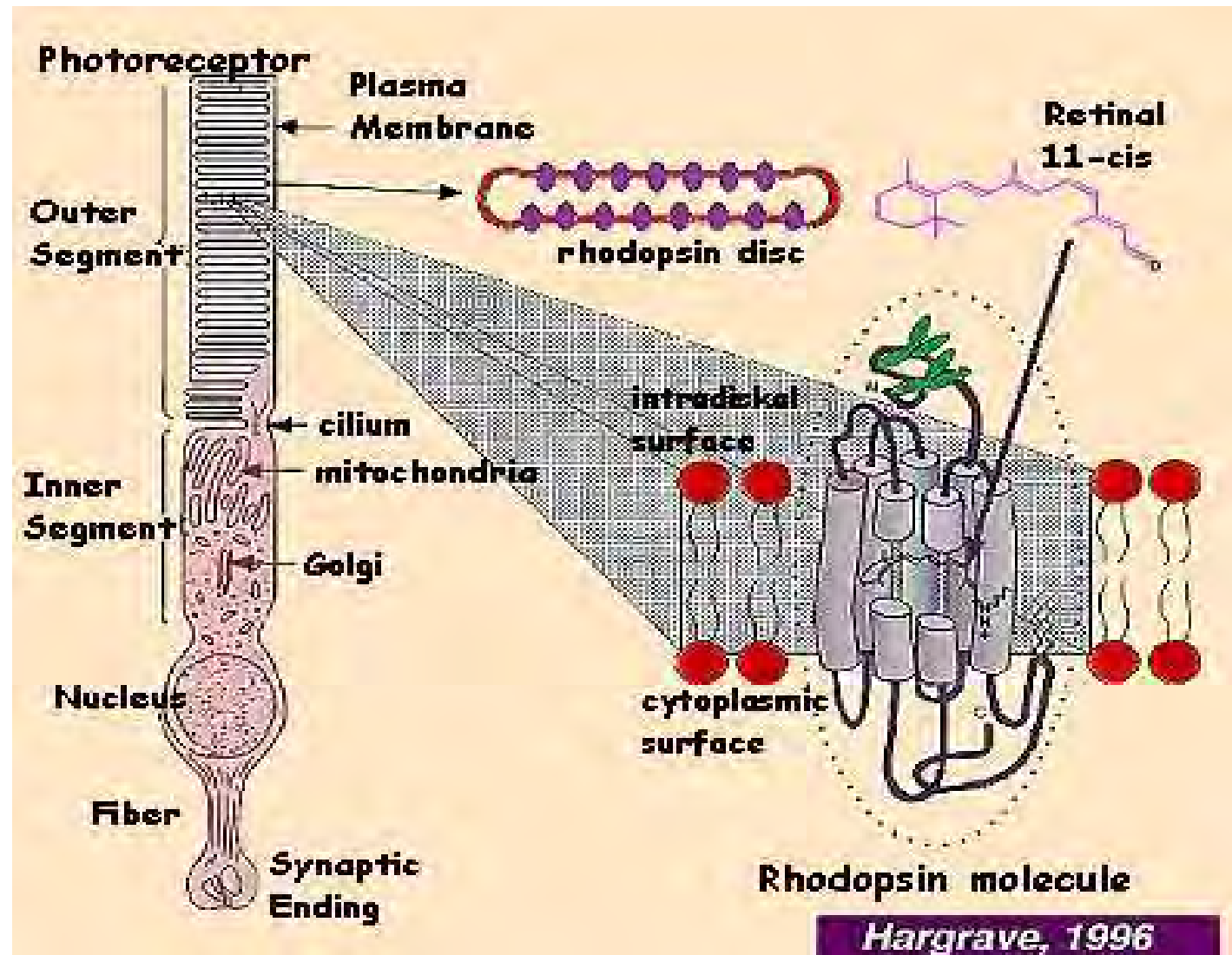
Vision Res. Vol. 7, 1967, pp. 829–836

# **Vision-I**

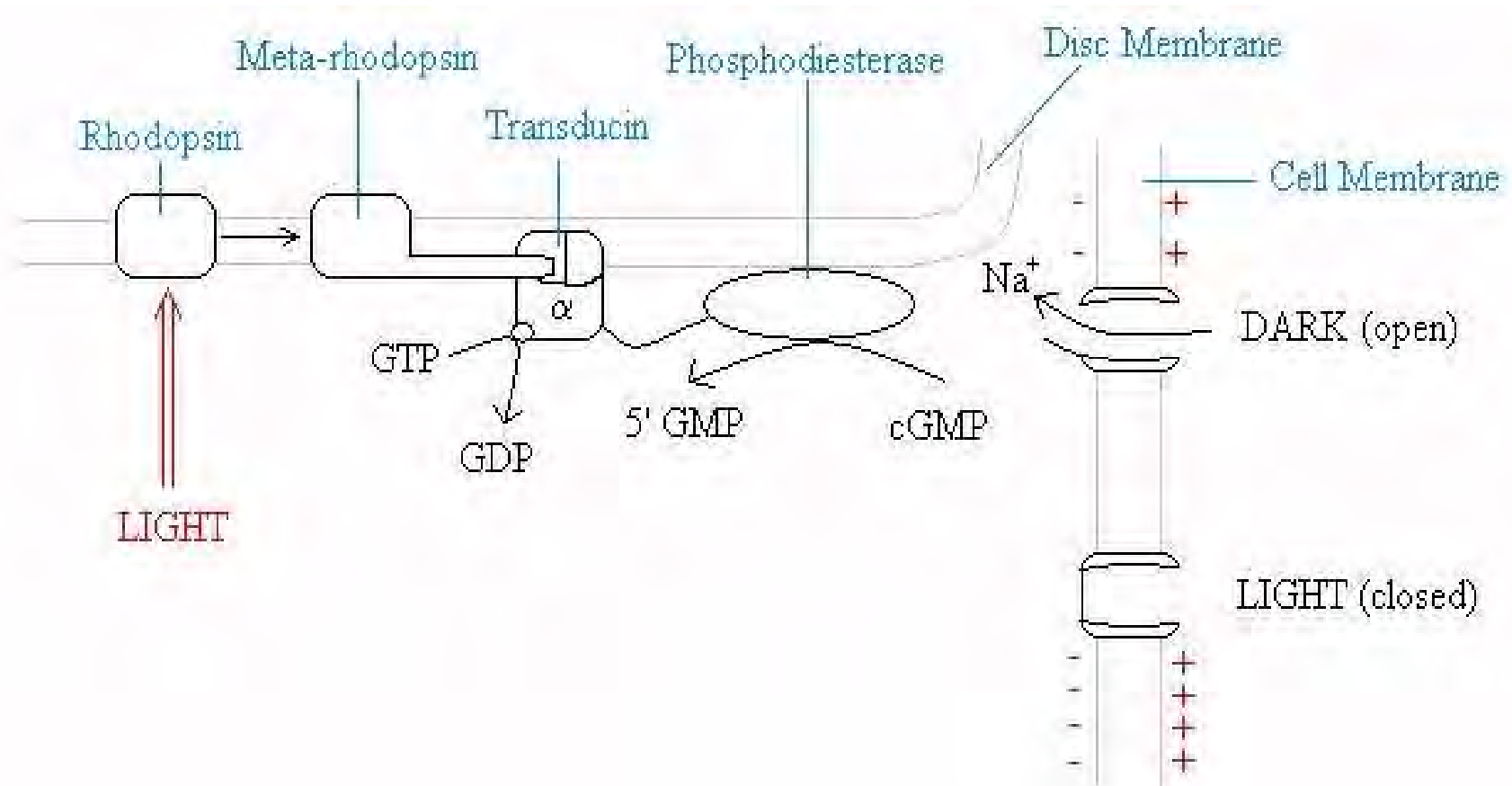
- **Geometry/Anatomy is part of the System Behavior of a Biological Process**
- **Intricate Geometry Fosters Novel Mathematics**



# Light Activation



# The Signaling Cascade on the Activated Disc



# The Cascade and Resulting Photocurrent

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$\{\text{Photon}\} \implies \text{R} \implies \text{R}^* \implies \text{G} \implies \text{G}^* \implies \text{PDE} \implies \text{PDE}^*$

depletion of cGMP  $\implies$  closing of the channels.

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The resulting *photocurrent* generated by the cGMP-gated channels is

$$J_{\text{cG}} = j_{\text{cG}}^{\text{max}} \frac{[\text{cGMP}]^{m_{\text{cG}}}}{K_{\text{cG}}^{m_{\text{cG}}} + [\text{cGMP}]^{m_{\text{cG}}}}$$

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**NOTE:** R, R\*, G, G\*, PDE, PDE\*, GC live on a "*surface*",  
whereas cGMP and  $\text{Ca}^{2+}$  live in a "*volume*".



## **Vision-II**

- **Combining Biophysics, Biochemistry and Geometry/Anatomy**
- **(subtitle) System Behavior**

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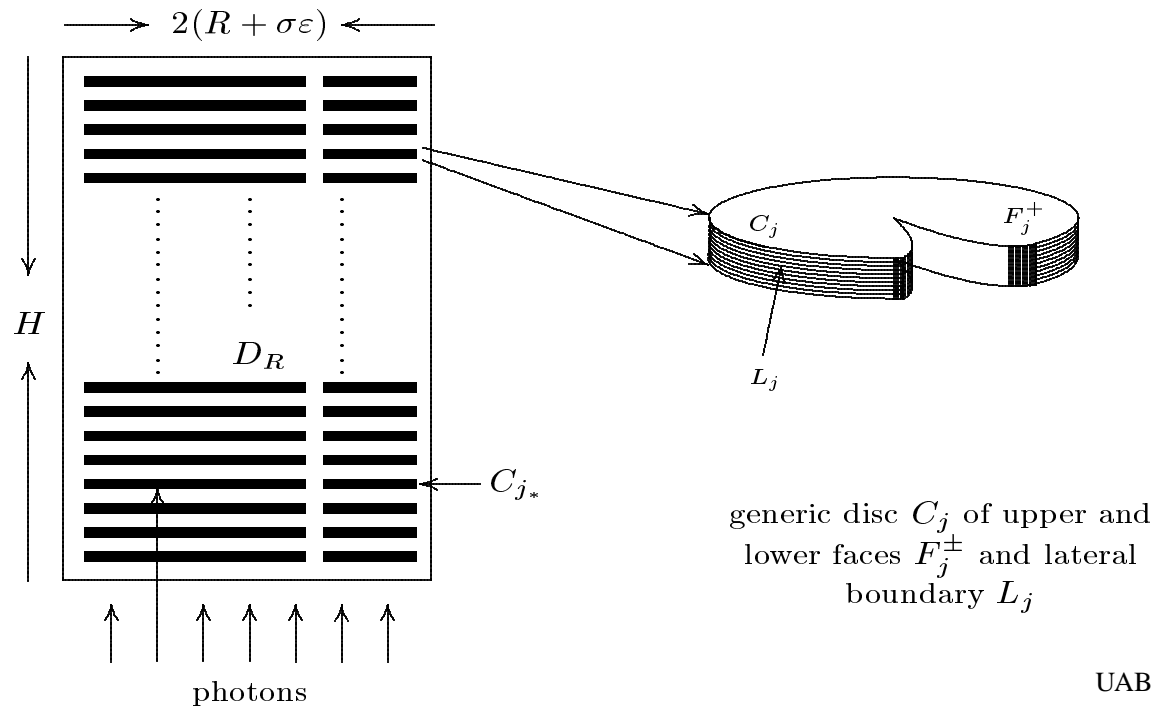
- **Homogenizing and Concentrating**

Reduce the problem to one with simpler geometry, still preserving the **same** features of the original one.

# The Limit with Constant Volume Fraction

The ratio of the volume occupied by the discs  $C_{\varepsilon,j}$  to the volume of  $\Omega - \mathcal{B}_\varepsilon$  is kept constant as  $\varepsilon \rightarrow 0$ .

$$\frac{\text{vol} \left( \bigcup_{j=1}^n C_{\varepsilon,j} \right)}{\text{vol}(\Omega - \mathcal{B}_\varepsilon)} = \mu_o$$



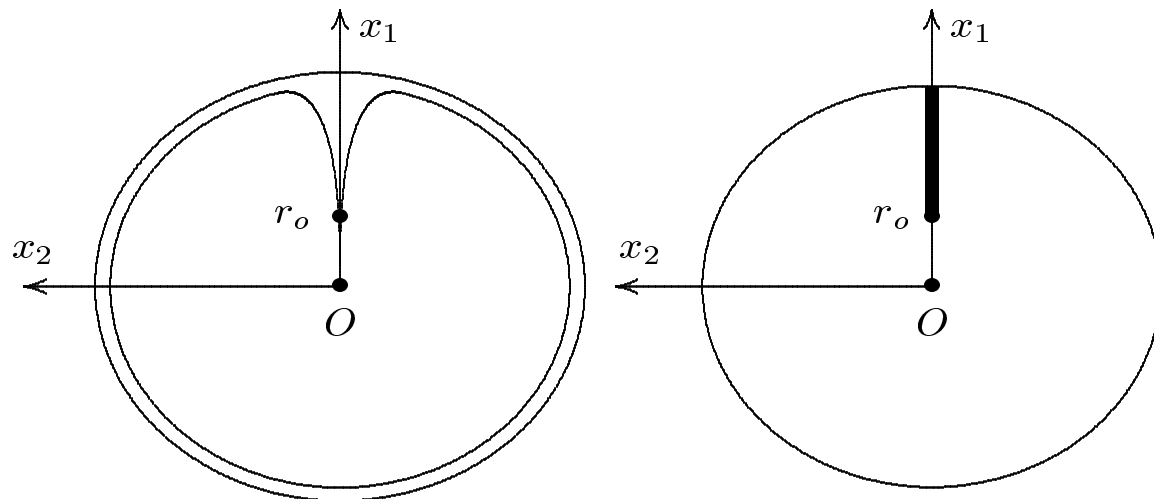


# The Formal Limiting Domains-I

$$\mathcal{V}_\varepsilon \longrightarrow \mathcal{V} = \{r_o < x_1 < R\} \times \{x_2 = 0\}$$

$$D_{R+\sigma\varepsilon} - \mathcal{V}_\varepsilon \longrightarrow D_R - \mathcal{V}$$

$$D_{R+\sigma\varepsilon} - D_R \longrightarrow \partial D_R$$

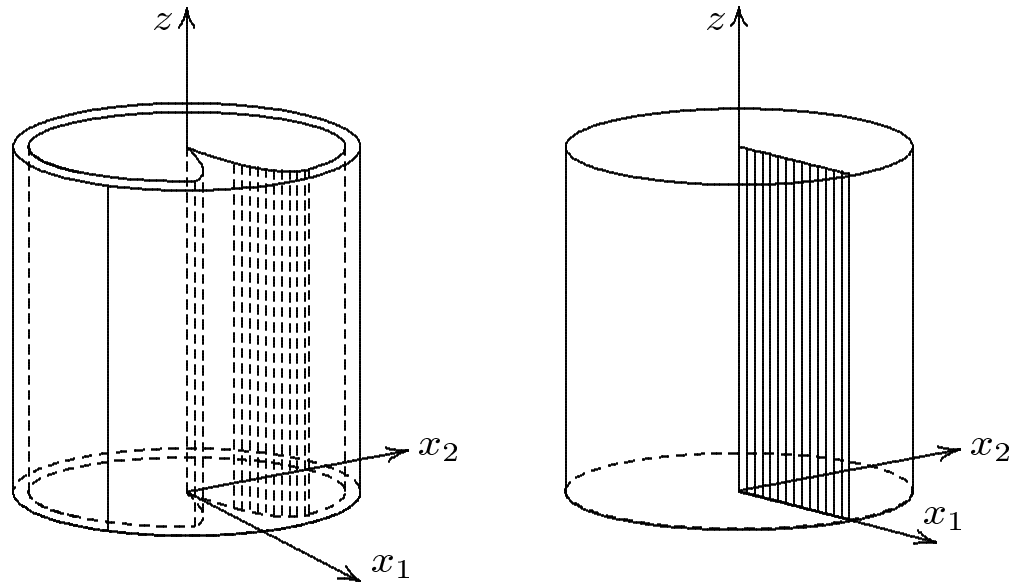


## The Formal Limiting Domains-II

$$S_\varepsilon = \{D_{R+\sigma\varepsilon} - D_R\} \times (0, H) \longrightarrow S = \partial D_R \times (0, H)$$

$$\mathcal{B}_\varepsilon = \mathcal{V}_\varepsilon \times (0, H) \longrightarrow \mathcal{V} \times (0, H)$$

$$\tilde{\Omega}_\varepsilon = \{D_{R+\sigma\varepsilon} - \mathcal{V}_\varepsilon\} \times (0, H) \longrightarrow \Omega = \{D_R - \mathcal{V}\} \times (0, H)$$



## Limiting Domains and Functions as $\varepsilon \rightarrow 0$

The approximating 2nd Messengers **cGMP** and **Ca<sup>2+</sup>** (say  $u_\varepsilon$  for **cGMP**) tend to 3 functions each satisfying a diffusion process in its domain of definition.

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- $u_{\mathcal{B}}$  defined in the limiting blade  $\mathcal{B}$ , and called the **limit in the incisure**



## Global Weak Form

$$\begin{aligned}
 & (1 - \mu_o) \left\{ \iint_{\Omega_T - \mathcal{B}_T} \{u_t \varphi + \nabla_{\bar{x}} u \cdot \nabla_{\bar{x}} \varphi + (u - f) \varphi\} dx dt \right\}_{\text{interior}} \\
 & + \sigma \varepsilon_o \left\{ \iint_{S_T} \{u_{S,t} \varphi + \nabla_S u_S \cdot \nabla_S \varphi\} d\eta dt \right\}_{\text{outer shell}} \\
 & + 2 \left\{ \iint_{\mathcal{B}_T} r \theta_{\varepsilon_o}(r) \{u_{\mathcal{B},t} \varphi + \nabla_{\mathcal{B}} u_{\mathcal{B}} \cdot \nabla_{\mathcal{B}} \varphi\} dr dz dt \right\}_{\text{incisure}} = 0
 \end{aligned}$$

valid for all testing functions  $\varphi \in W^{1,2}(\bar{\Omega}_T)$  roughly speaking in the dual classes of  $u$ ,  $u_{\mathcal{B}}$  and  $u_S$ .

## **Vision-III**

- **Novel Mathematics**
- **(subtitle) Partial Differential Equations in Fractured Domains or Domains with Spikes**

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$$f([cGMP]) = \frac{1}{2} \left\{ \beta_{\text{dark}}[cGMP] - \frac{\alpha}{1 + ([Ca^{2+}]/K_{\text{cyc}})^{m_{\text{cyc}}}} \right\} \\ - \delta_{z_*} k_{\sigma; \text{hyd}}^* [PDE^*]_{\sigma} [cGMP]$$

Here  $[cGMP]$  and  $[Ca^{2+}]$  are unknowns to be determined by the system of equations.  $[PDE^*]$  is a **datum generated by the activation cascade**. More on this later.

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## VISION: Inform Mathematics with Biochemistry

# Simulated and Experimental Current Drop $1 - J/j_{\text{dark}}$

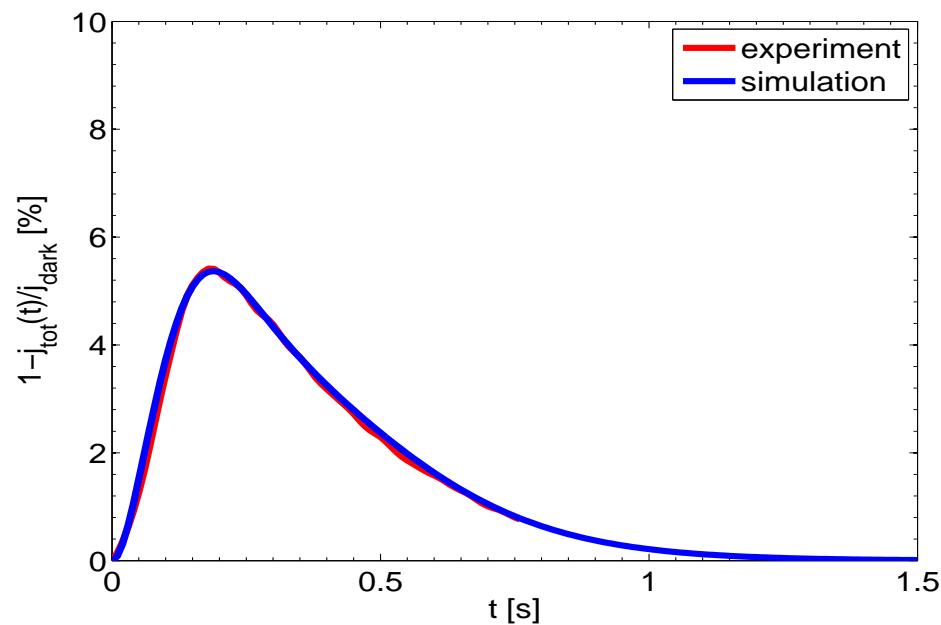


## Simulated and Experimental Current Drop $1 - J/j_{\text{dark}}$

- Experimental data for mouse SPR provided by F. Rieke (Seattle).

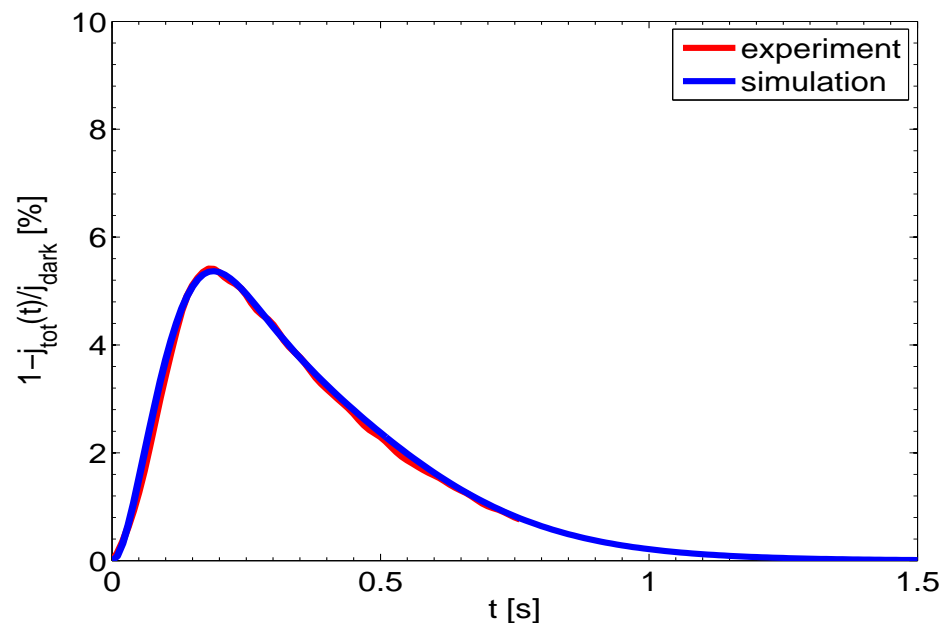
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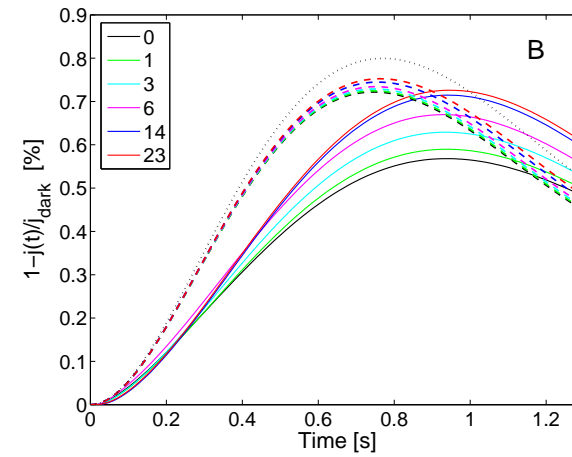
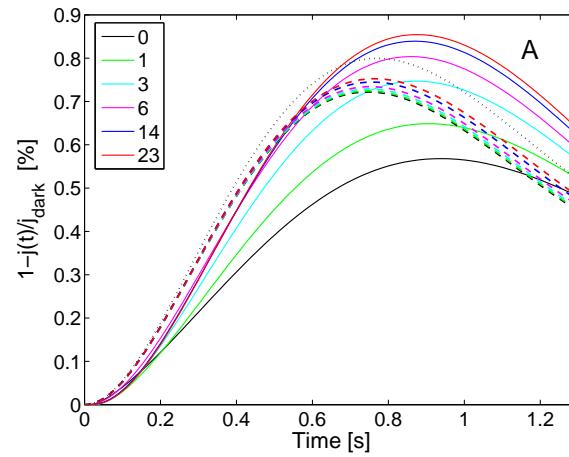
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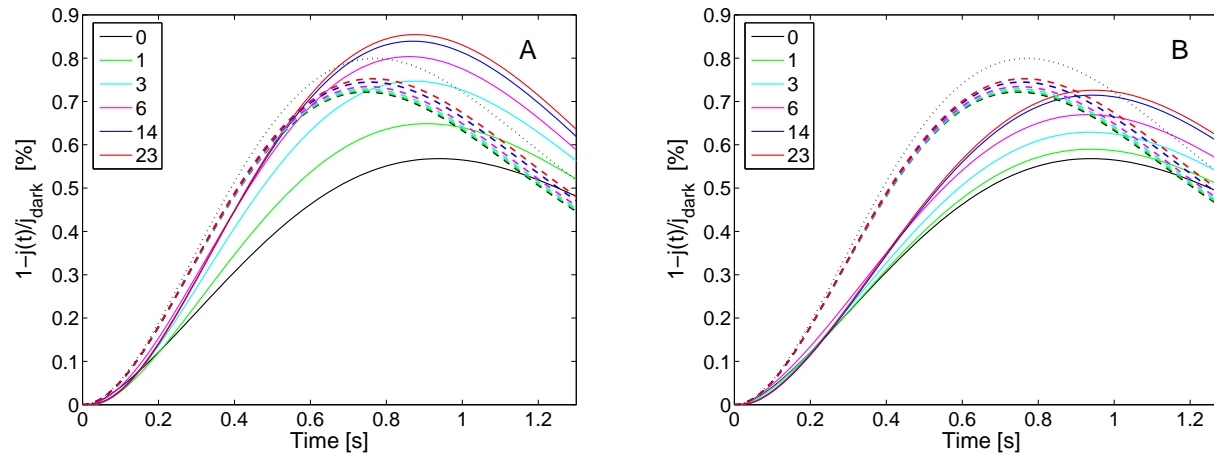
- Full Model** (non homogenized) 8sec simulation on 20 processors of IBM series cluster (Oak Ridge Labs) took 57 hours. Same 8sec simulation with the **HM** (homogenized model) 8sec simulation took 50 seconds of an Intel Xeon

# Single Photon. Current as a Function of the Incisures (Salamander)

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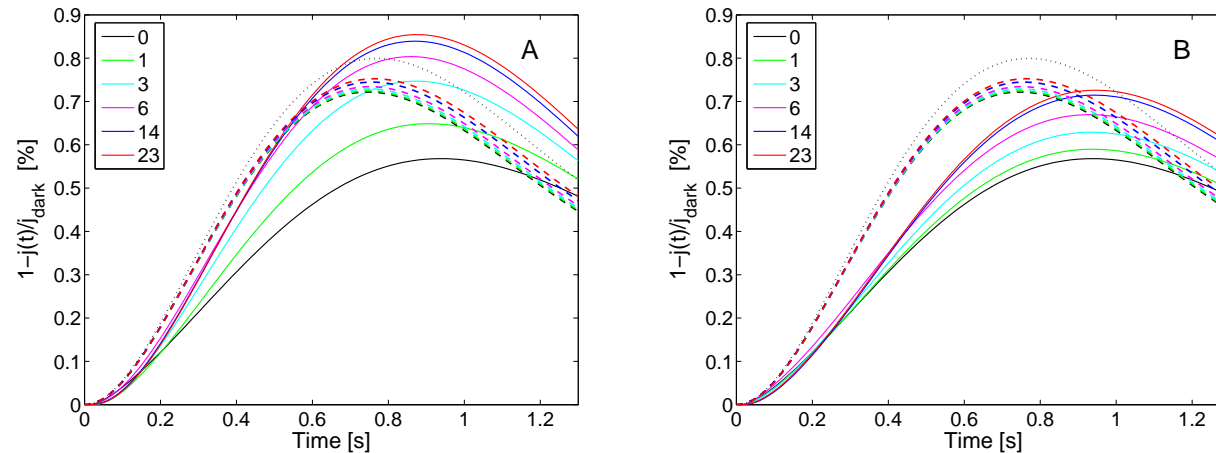


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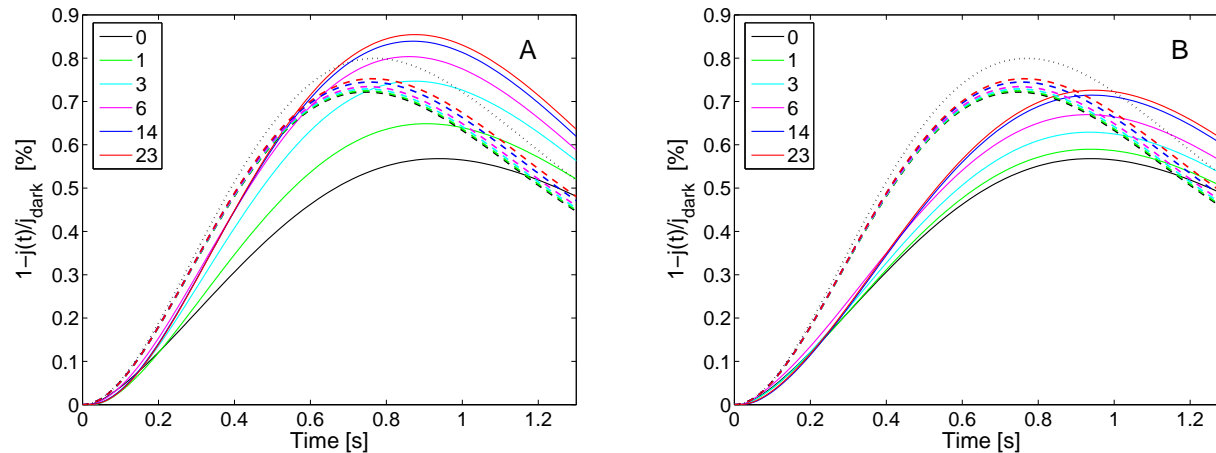
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- **Panel B:** Incisures with equal area but half the length and double the width of those in the left figure (wide and short, with tip away from activation site).



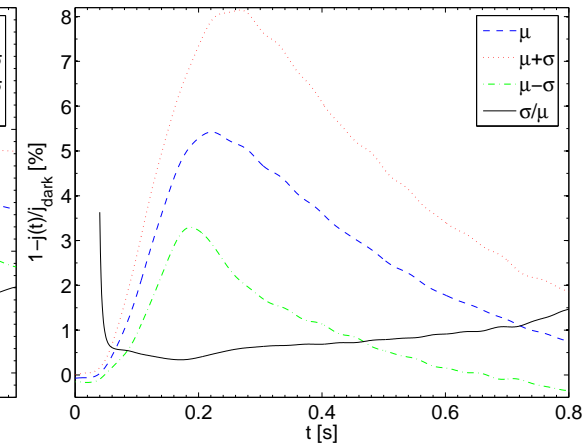
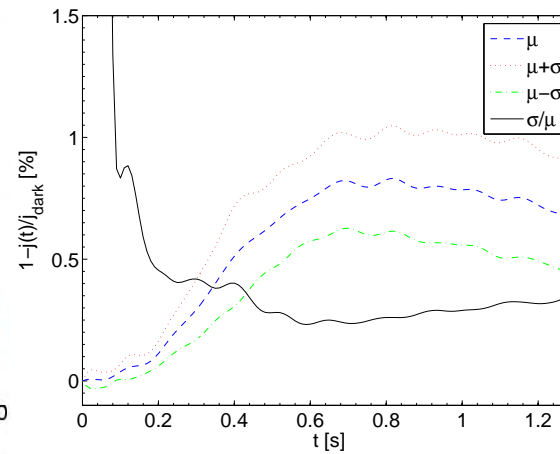
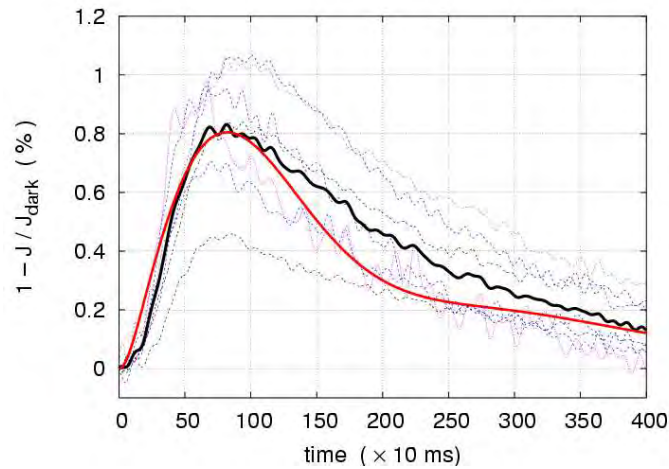
## **Vision-IV**

- **Numerical Analysis, Programming, Simulating**
- **(subtitle) Capability of Replacing Wet-Bench Experiments with Virtual Experiments**

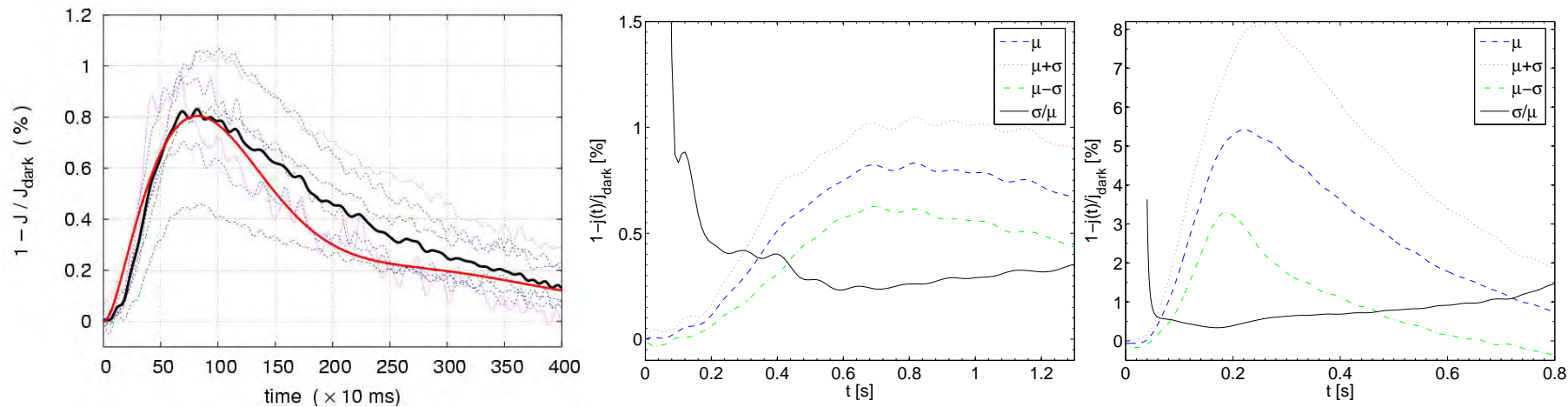


# **The Activation/Deactivation Cascade, Variability and Fidelity of the SPR**

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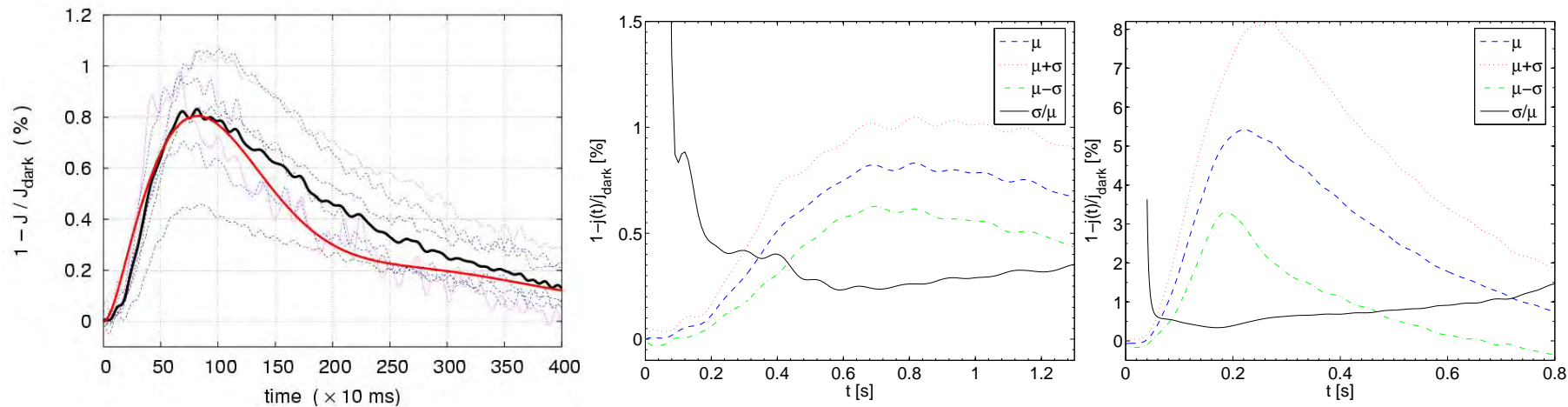


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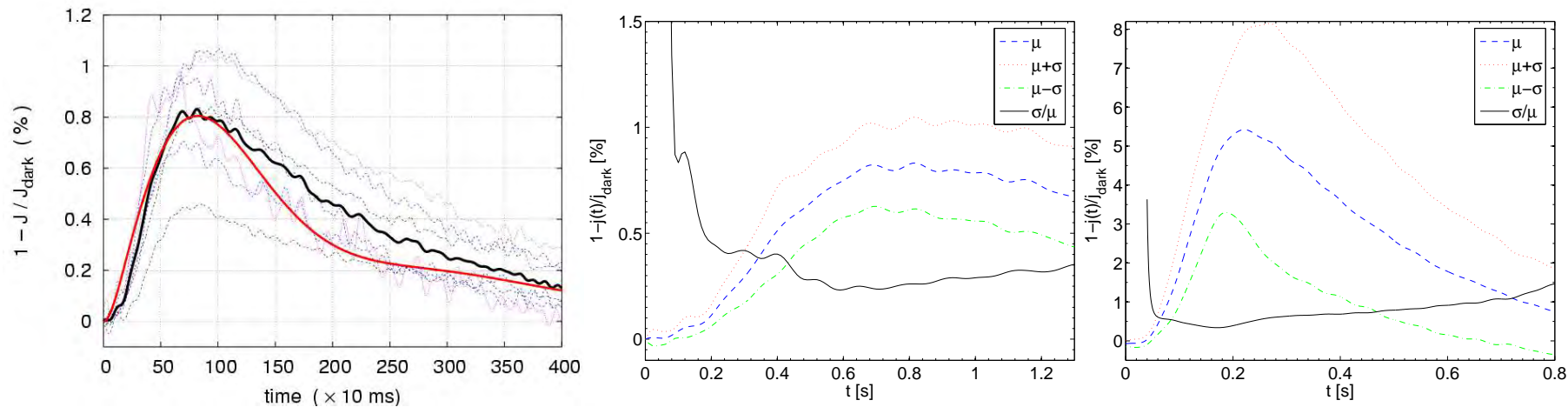
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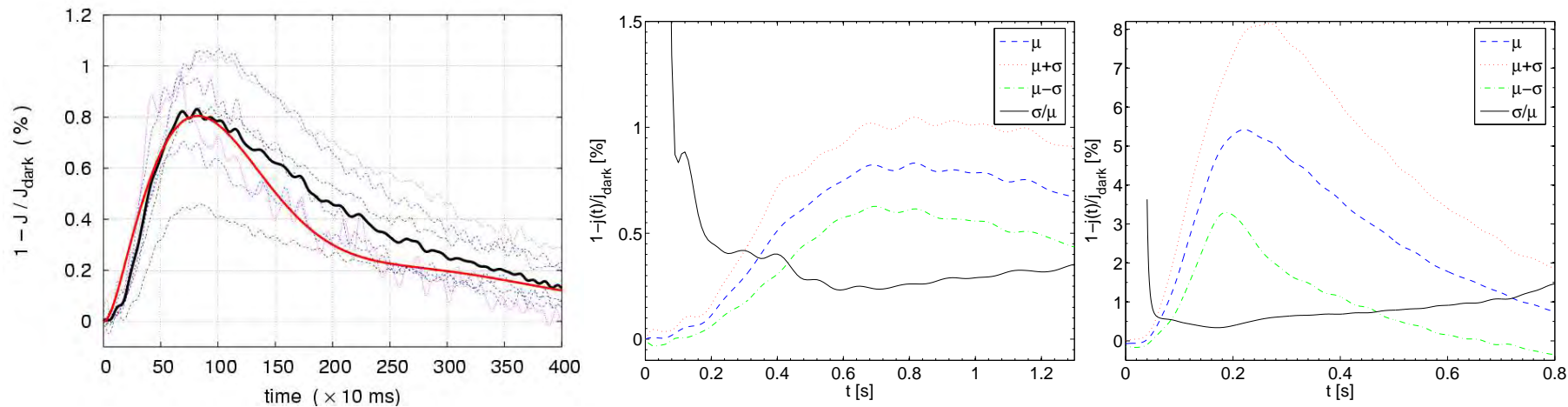
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- **Right Panel:** Experimental CV for the Mouse SPR (Rieke)

# The Activation Cascade and Quenching



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- A molecule of rhodopsin, upon activation at some random location  $\mathbf{x}_o$ , follows its random path  $t \rightarrow \mathbf{x}(t)$  in  $D_{\text{eff}}$  and becomes inactivated abruptly after a random time  $t_R$ , of average  $\tau_R$ .

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- The transitions from the state  $j$  to  $(j + 1)$  occurs at some random time  $0 < t_j \leq t_n = t_R$  where  $t_o = 0$ .

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- **Pugh, E. N. Jr., Variability of Single Photon Responses: A Cut in the Gordian Knot of Rod Phototransduction ? *Neuron*, 1999, 23:205–208**

## **Vision-V**

- **Interplay Between Stochastic and Deterministic Processes. Both are Present Essentially Always**
- **(subtitle) Analyze Randomness by Knowing the Biochemistry**
- **(subtitle) Again System Biology**

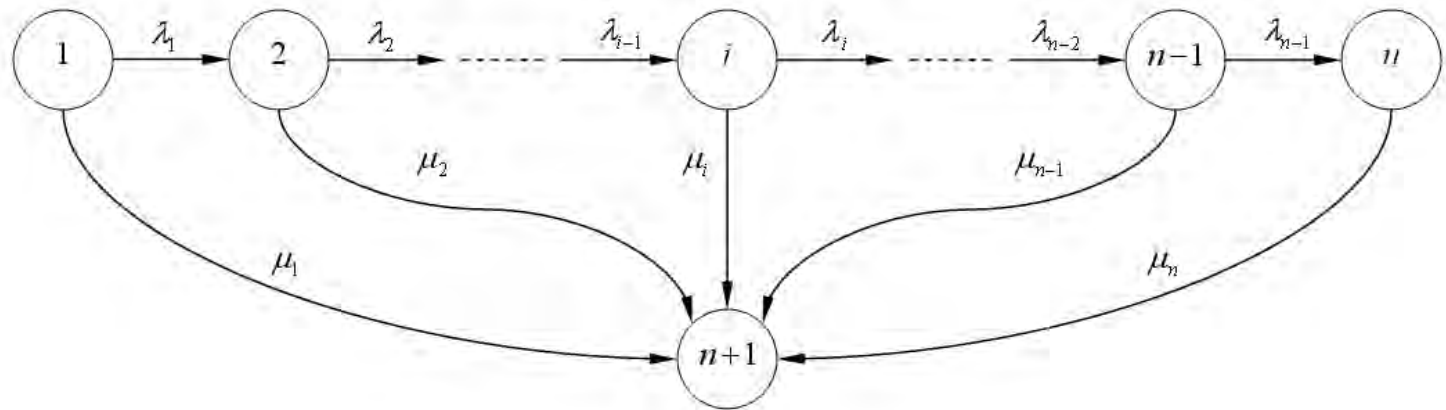
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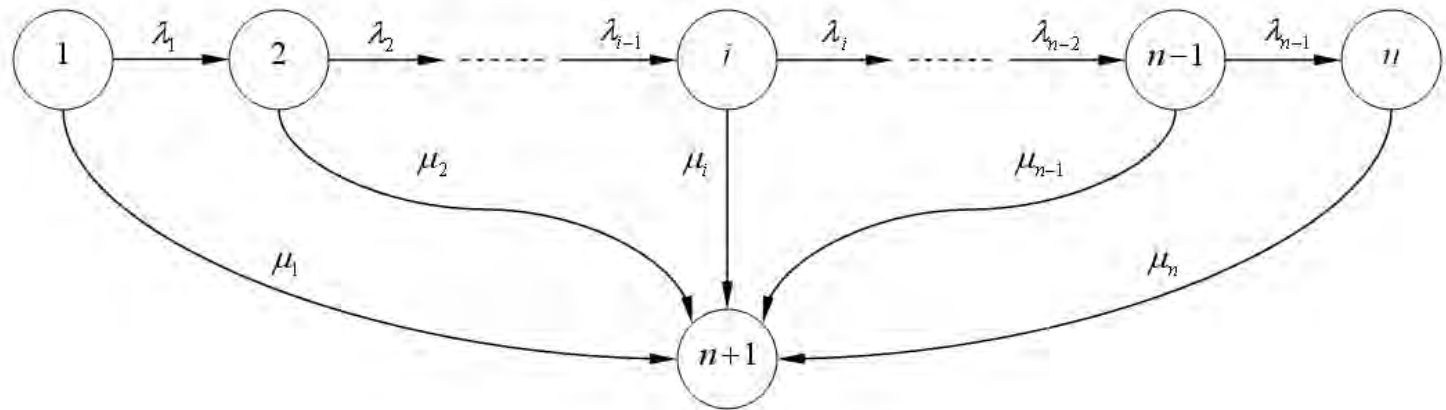
- Phosphorylation of rhodopsin by RK is a cooperative process, i.e., the incorporation of one or two phosphates increases the probability of further phosphorylation.
- The catalytic activity of  $R^*$  decreases with increasing levels of phosphorylation, at a rate of about 12% for each additional level of phosphorylation.
- **V. Gurevich**, The Selectivity of Visual Arrestin for Light-Activated Rhodopsin is Controlled by Multiple Non-Redundant Mechanisms, *J. Biol. Chem.*, **1998**, 15501–15506.

# Continuous Time Markov Chain (CTMC): Variability and Biochemistry

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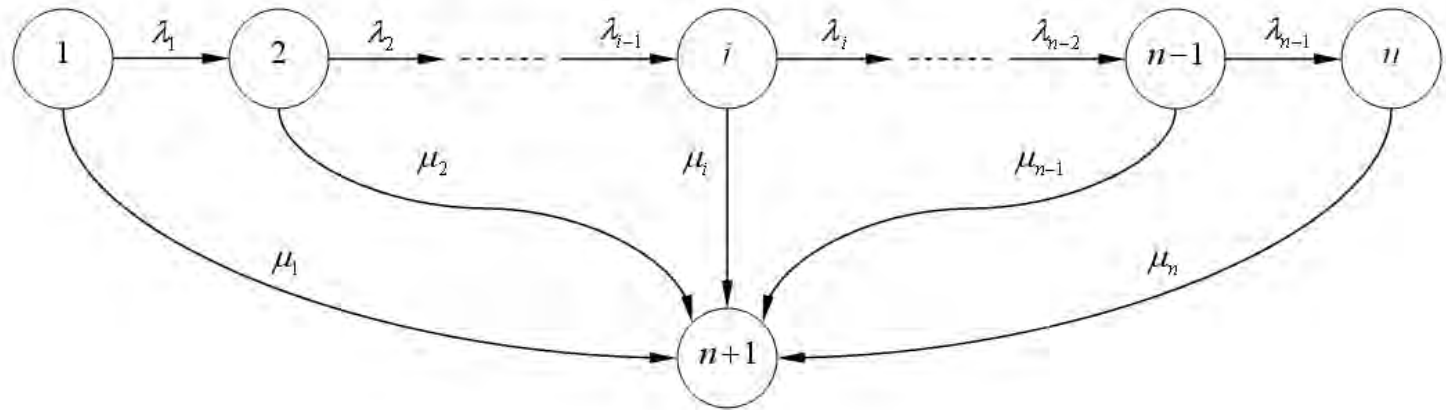


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Here  $\lambda_j$  are affinities to phosphorylation and  $\mu_j$  are affinities to Arrestin (Arr). Bernoulli process with probabilities  $\frac{\lambda_j}{\lambda_j + \mu_j}$  and  $\frac{\mu_j}{\lambda_j + \mu_j}$ .

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**The  $\lambda_j$  and  $\mu_j$  represent the Biochemistry of Deactivation.**



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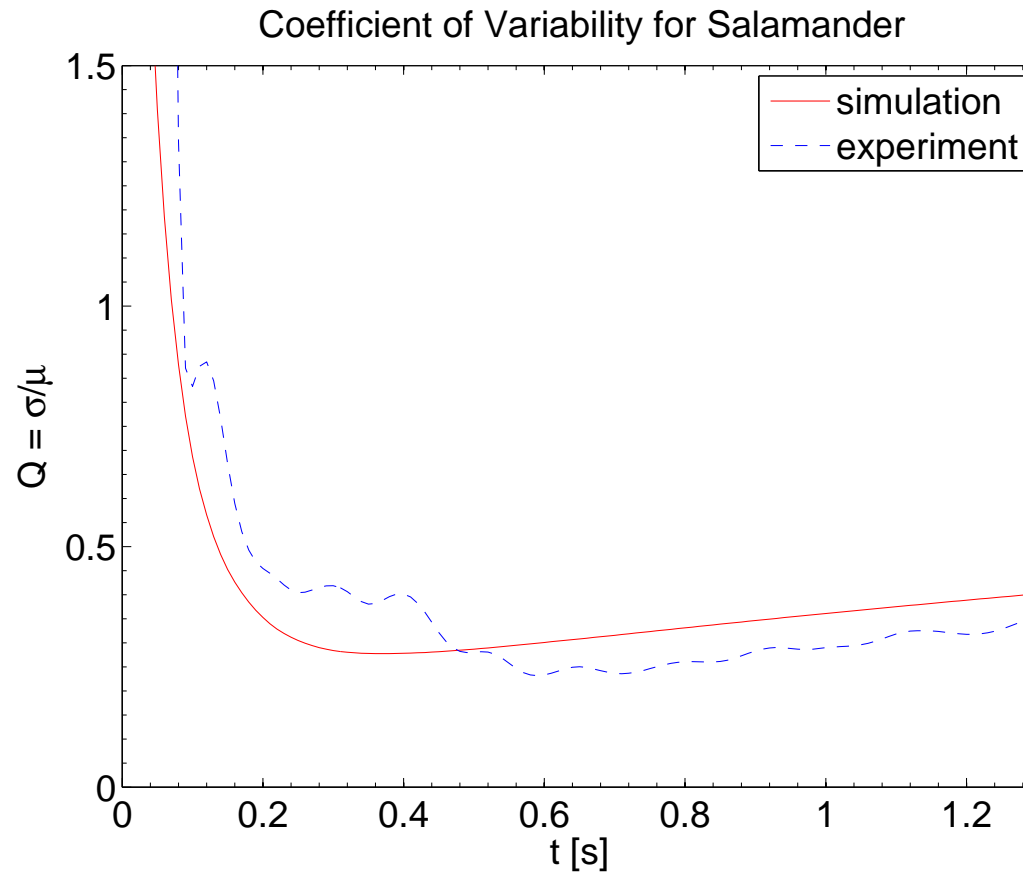
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$E^{**}(t)$       total activity of  $[PDE^*]$  up to time  $t$

Main issue is:

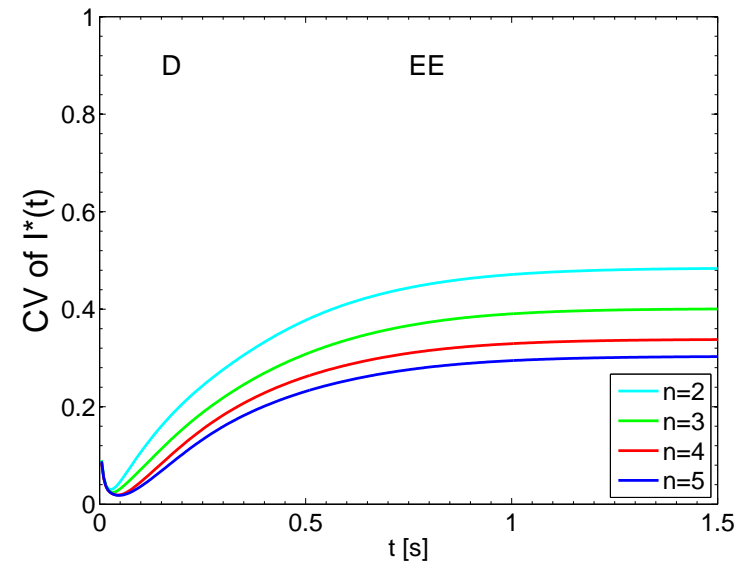
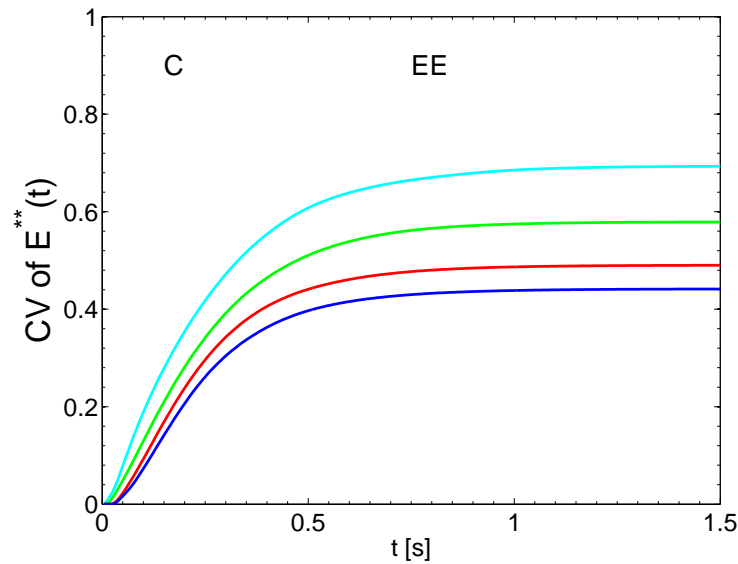
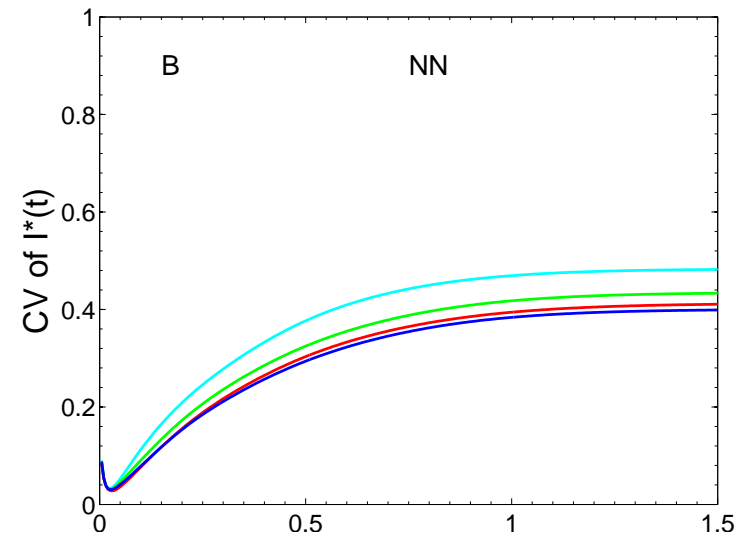
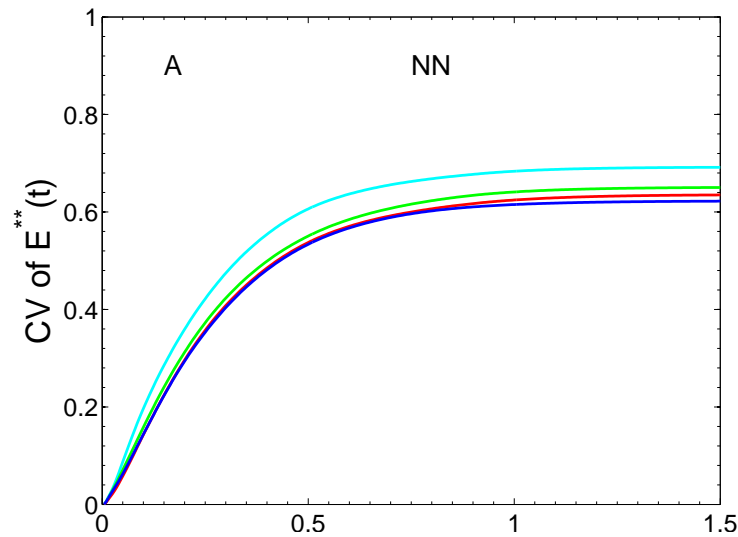
- Why is the CV of the photocurrent low ?

# Simulated and Experimental CV(= $\sigma/\mu$ ) of Current Drop

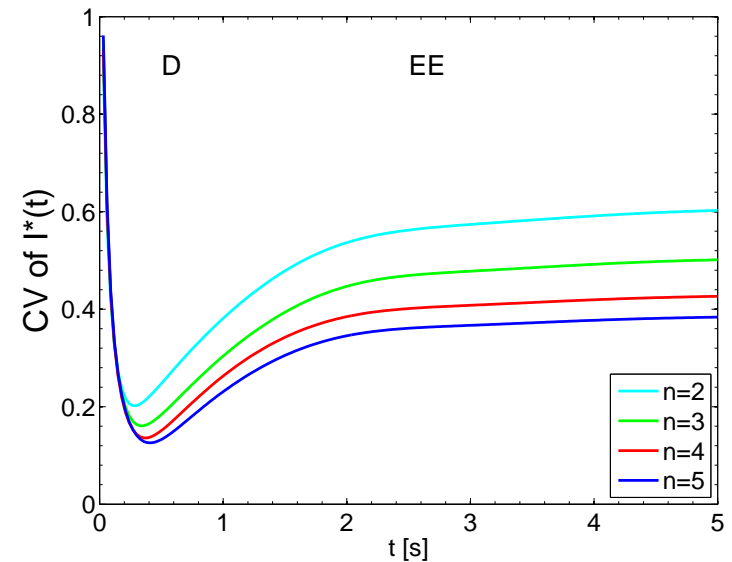
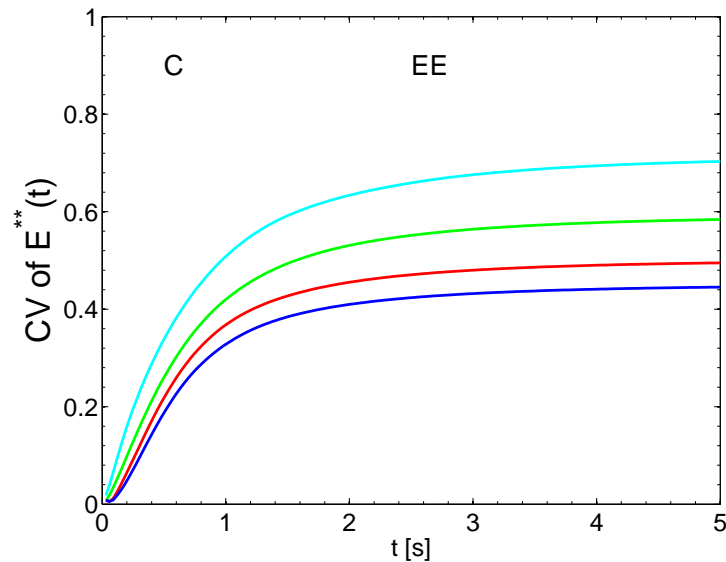
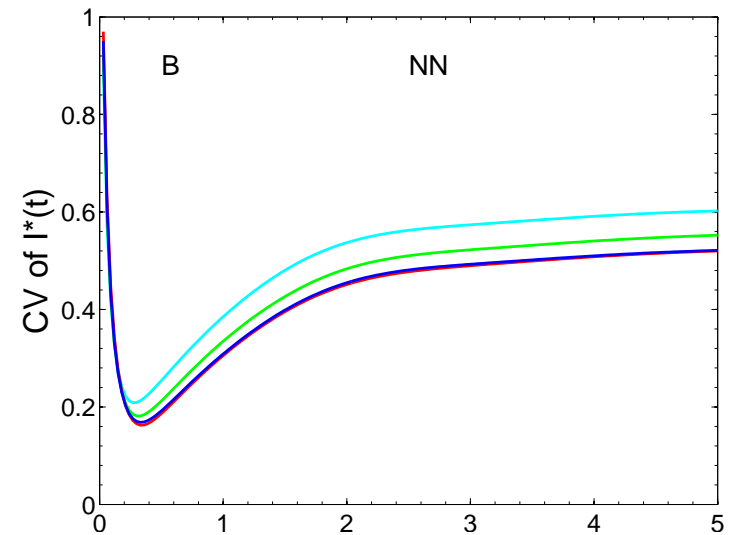
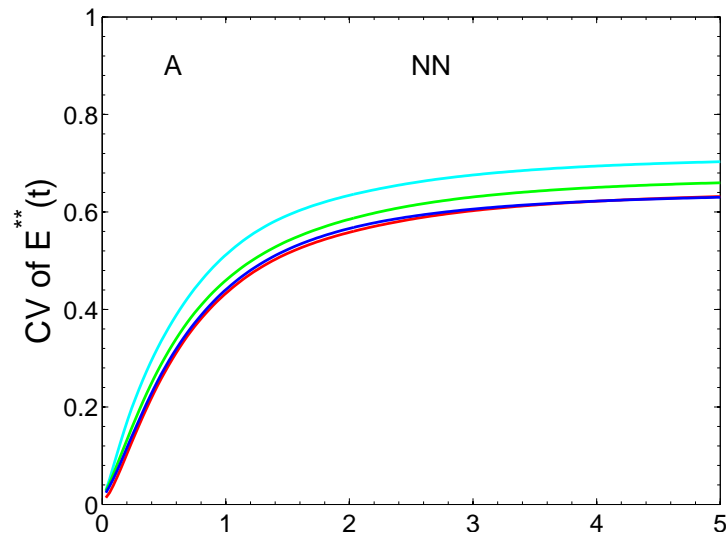


Initial instability due to Incisures; up to 32 for the Salamander, and only 1 for Mouse

# Simulated $CV(= \sigma/\mu)$ . Mouse



# Simulated $CV(= \sigma/\mu)$ . Salamander



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  - CV is suppressed by the diffusion of the second messengers cGMP and  $\text{Ca}^{2+}$

## Some Bibliography

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- **Web site:** <http://abcd.math.vanderbilt.edu/~signaltr/>

## **Vision-VI**

- **Parameter Determination is an Art of its Own**
- **However ... Beware of Parameter Fitting**