EXISTENCE RESULTS FOR SOME CLASSES OF INTEGRODIFFERENTIAL EQUATIONS OF GURTIN-PIPKIN TYPE

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ABSTRACT — Integro-differential equations of Gurtin-Pipkin type play an important
in studying various practical problems. In particular, they have been used to study the
heat conduction in materials with memory, the sound propagation in viscoelastic media,
or in homogenization problems in perforated media (Darcy’s Law). The main purpose of
this talk consists of studying the existence of bounded and asymptotically almost periodic
solutions to the class of integro-differential equations of Gurtin-Pipkin type given by

$$\frac{d^2 u}{dt^2} + A^2 u - \int_{-\infty}^{t} g(t-s)A^2 u(s)ds = f(t, u), \quad t > 0$$

with initial conditions

$$u(-t) = u_0(t), \quad t \geq 0 \quad \text{and} \quad u'(0) = u_1,$$

where \(A : D(A) \subset H \mapsto H\) is a positive self-adjoint operator on a separable Hilbert space
\(H\), which is bounded below, that is, there exists a constant \(\omega > 0\) such that

$$\|Au\| \geq \omega \|u\| \quad \text{for all} \quad u \in D(A),$$

the function \(f : [0, \infty) \times H \mapsto H\) is asymptotically almost periodic in the first variable
uniformly in the second one, and the non-increasing differentiable relaxation function \(g : [0, \infty) \mapsto [0, \infty)\) satisfies the following assumptions,

(A.1) \(g(0) > 0;\)

(A.2) \(1 - \int_0^{\infty} g(s)ds = \beta > 0;\) and

(A.3) there exists a positive constant \(\xi\) such that

$$g'(t) \leq -\xi g(t) \quad \text{for all} \quad t \geq 0.$$

To illustrate our abstract results, two examples will be discussed one of which is the
well-known Kirchoff plate equation.