CALCULUS I

Final Exam, December 14, 2016

Name (Print last name first): ....

Show all your work, justify and simplify your answer!

No partial credit will be given for the answer only!

PART I

You must simplify your answer when possible but you don't need to compute numbers: $\mathbf{e^6 \sin(12/5) + 8}$ is a fine answer.

All problems in Part I are 6 points each.

(1) Use the **definition** of the derivative to show that the the derivative of the function $f(x) = x^2 + 3$ is $f'(x) = 2x$.

(2) Find the derivative of the function $f(x) = x \sin(x)$.

(3) Find the derivative of the function $f(x) = \frac{x}{\cos(x)}$.

(4) Find the derivative of the function $f(x) = \sin(x^3)$.

(5) Evaluate $\int x(x^3 + x)\, dx$.

(6) Evaluate $\int x^2 \sin(x^3 + 4)$.

(7) Evaluate $\displaystyle\int \frac{x^3 + x}{x^2}\, dx$

(8) Find the linearization of the function $f(x) = \ln(x)$ at $a = 1$ and use it to approximate the value $f(1.1) = \ln(1.1)$.

(9) Find the derivative of the function $F(x) = \int_2^x \cos(t^3)\, dt$.

(10) Use a Riemann sum with $n = 2$ terms and the midpoint rule to approximate the value of $\int_2^3 \cos(x^3)\, dx$.

PART II

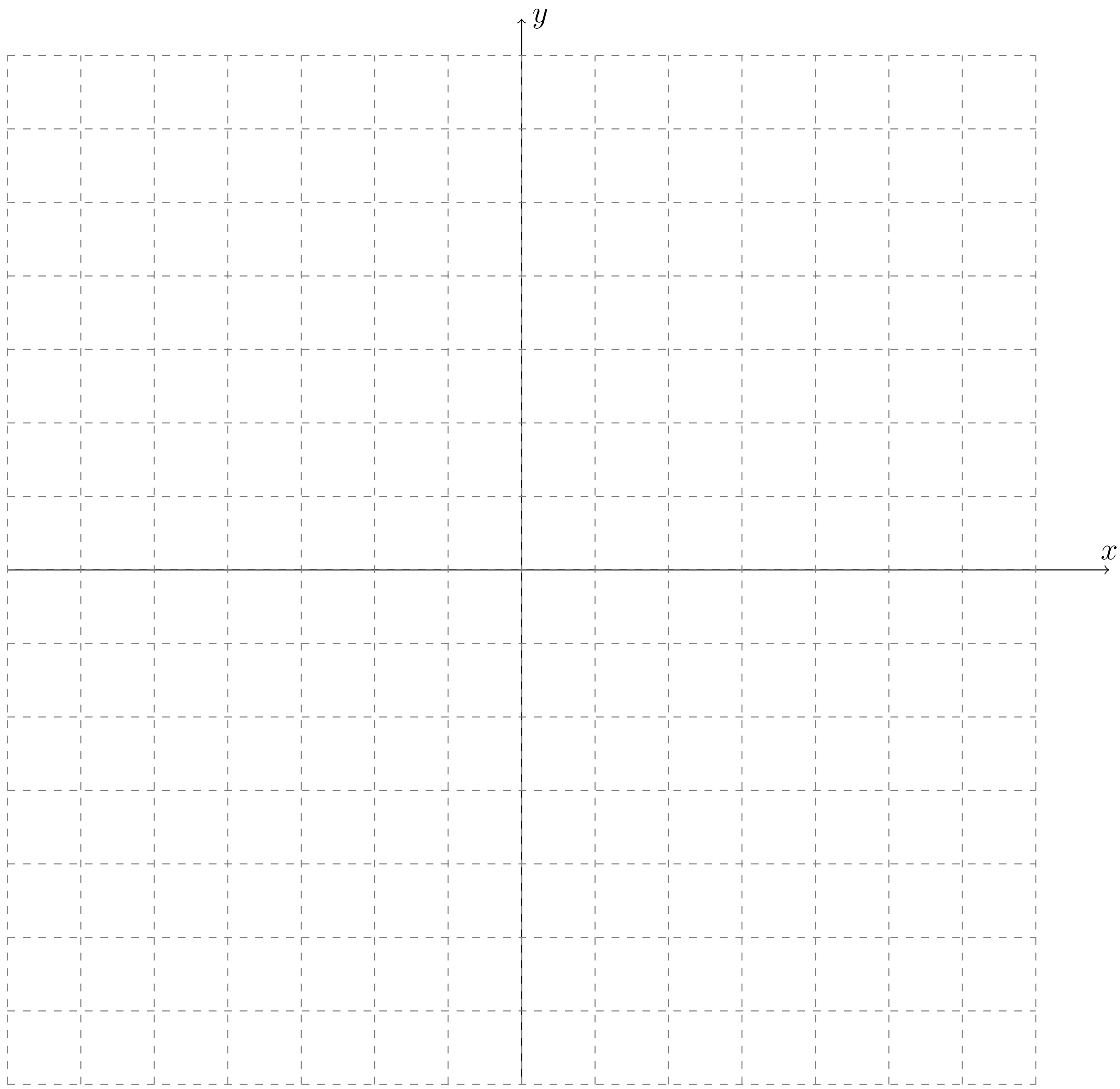
(1) **[10 points]** Show that the equation $f(x) = x^3 - 9 = 0$ has exactly one solution. Then use Newton's method with $x_1 = 2$ to compute the second approximate solution.

(2) **[14 points]** Use calculus to graph the function

$$f(x) = \frac{2x^2+1}{x^2-1}.$$

Find:
(a) all x and y-intercepts,
(b) horizontal and vertical asymptotes (if any),
(c) critical numbers, where the function is in/de-creasing, and
(d) absolute/local maxima/minima (if any).

Draw your graph below but do your work for (a)–(d) on the next page.



Work for problem II–(2).

(3) **[8 points]** Find the absolute maximum and minimum of the function $f(x) = x\,(1+x)^3$ on the interval $[-2, 2]$.

(4) **[8 points]** Find the dimensions of an oil barrel with volume $V = 1\ m^3$ of minimal cost if the martial for side and bottom costs \$10 $/m^2$ and the material for the top costs \$2 $/m^2$.

Hint: the volume of a barrel with radius r and height h is $V = \pi r^2 h$, the area of top and bottom is πr^2 and the area of the side is $2\pi r h$.

Scratch paper