

MA 125 CW, CALCULUS I
September 26, 2016

Name (Print last name first):

TEST I

Show all your work! No partial credit will be given for the answer only!

PART I

Part I consists of questions. Clearly write your answer in the space provided after each question. Show all of your your work!

All problems in Part I are 7 points each

Evaluate the following limits.

Question 1

Use the **definition** of the derivative to show that the derivative of $y = f(x) = x^2$ is $f'(x) = 2x$.

Question 2

Find the derivative of $f(x) = x(x^2 + \sqrt{x})$

Answer:

Question 3

Find the derivative of $y = f(x) = x \sin(x)$.

Answer:

Question 4

Find the derivative of $y = f(x) = \frac{x^2}{x+1}$.

Answer:

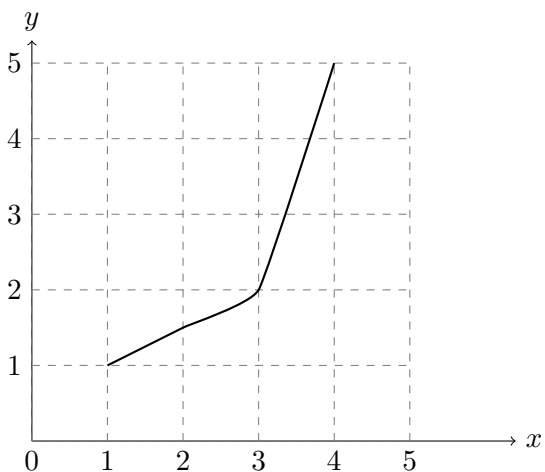
Question 5

Find the equation of the tangent line to the graph of $y = f(x) = 2x^3 + x$ at the point $x = 1$.

Answer:

Question 6

Using the graph of the function $y = f(x)$ below estimate: $f(2)$ and the derivative $f'(2)$.



Answer:

Question 7

If $P(x)$ is the profit of selling x -items. What is the meaning of the derivative $P'(x)$?

Question 8

Show that the equation $f(x) = x^7 + x + 3 = 0$ has at least one solution; find an interval of length one which must contain a solution.

PART II

Part II consists of 4 problems. You must show correct reasons to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit.

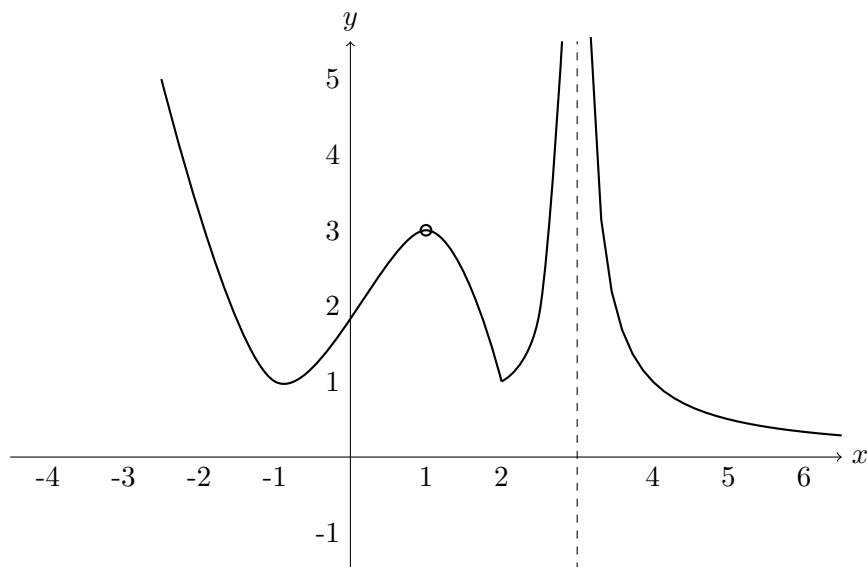
Problem 1 (12 points)

Suppose that $S(t) = 2t^3 + 3t^2 - 12t$ m. is the position of a particle at time t (in seconds) on a line. Find:

- (a) the velocity at time $t = 1$.
- (b) The displacement from $t = 0$ to $t = 3$.
- (c) The total distance travelled between $t = 0$ and $t = 3$.

Problem 2 (10 points)

Given the graph of the function $y = f(x)$ below find:



1. $\lim_{x \rightarrow 1^-} f(x) =$
2. $\lim_{x \rightarrow 1^+} f(x) =$
3. $\lim_{x \rightarrow 1} f(x) =$
4. $\lim_{x \rightarrow 3^-} f(x) =$
5. $\lim_{x \rightarrow 3^+} f(x) =$
6. $\lim_{x \rightarrow 3} f(x) =$
7. $\lim_{x \rightarrow \infty} f(x) =$
8. $\lim_{x \rightarrow -\infty} f(x) =$
9. State all intervals on which $f(x)$ is continuous.
10. State all intervals where $f(x)$ is differentiable.

Problem 3 (12 points)

Evaluate the following limits:

a) $\lim_{x \rightarrow \infty} x^3 + x$

b) $\lim_{x \rightarrow \infty} x^3 - x$

c) $\lim_{x \rightarrow \infty} \frac{\tan(x)}{x}$

Problem 4 (10 points)

Define a function

$$f(x) = \begin{cases} \frac{-x^3+5x^2}{x-5}, & x \neq 5, \\ k, & x = 5, \end{cases}$$

- a) (8 points) Find a value of k so that $f(x)$ is a continuous function for all real numbers x .
- b) (2 points) For the value of k you found, is $f(x)$ differentiable at $x = 5$? Briefly explain.