

TEST 1

Duration 70min;

Make sure to show all your work and underline the final results of each problem. Write your name on this sheet and use it as a cover page when you turn in your work. Do not write your results on this paper. Good luck!

1. The graph of f consists of two straight lines and a semi circle. Use it to evaluate each integral.

$$(a) \int_3^4 f(x) dx, \quad (b) \int_{-1}^2 f(x) dx, \quad (c) \int_0^4 f(x) dx$$

2. (a) Use the properties of integrals to verify that

$$\int_1^3 \frac{1}{\ln(x) + 2} dx \leq 1$$

(b) Derive a good lower bound for integral in a similar way as the upper bound is derived. (E.g. -1 is a correct lower bound but not good enough.)

3. Write out the form of the partial fraction expansion of the function. Do not determine the numerical values of the coefficients.

$$(a) \frac{2x - 7}{(x - 1)^2(x + 1)} \quad (b) \frac{1 + 3x - x^2}{(x^2 - 14x + 50)x}$$

4. Evaluate the following integrals

$$(a) \int_1^2 u^3 du$$

- (b) $\int \frac{5-x}{\sqrt{x}} dx$
- (c) $\int \frac{1}{5x-2} dx$
- (d) $\int_{-2}^2 \frac{x^2 \sin(x)}{1+x^4} dx$
- (e) $\int (\cos x)^4 (\sin x)^3 dx$
- (f) $\int x^2 (1-x^3)^5 dx$
- (g) $\int t^{1/2} \ln(t) dt$
- (h) $\int \frac{x^4}{x^2+1} dx$
- (i) $\int \frac{1}{x^2-1} dx$

5. Find the derivative of the function

$$g(x) = \int_0^{\sin(x)} \frac{2}{t^2 + \ln(t+2)} dt$$

Bonus. Prove the following statement. If

$$\int_{-x}^x f(t) dt = 0 \quad \text{for all } x > 0$$

then f is an odd function: $f(-x) = -f(x)$.