MA 126, Calculus 2, UAB, Fall 2004.

TEST 2

Duration 75 min, Max. Points: 45

Make sure to show all your work and underline your final results. Write on these sheets or use extra paper if needed. Each subproblem is worth 3 points. Good luck!

1. Determine whether the given improper integral is convergent or divergent. To this end you may or may not need to evaluate it.

(*a*) $\displaystyle\int_{-7}^{\infty} \frac{1}{e^x}\,dx$

(*b*) $\displaystyle\int_{1}^{\infty} \frac{(\sin x)^2}{x^2\sqrt{x}+3}\,dx$

(c) $\displaystyle\int_2^\infty \frac{1}{\sqrt{x^2-1}}\,dx.$

2. Is the given SEQUENCE convergent or divergent? Justify your answer! If it is convergent find the limit.

(a) $a_n = \dfrac{1-7n+n^2}{3n^2-8n}$

(b) $a_n = \dfrac{4^n}{3^{n+5}}$

(c) $a_n = \dfrac{\ln n}{n^{2/3}}$

(d) $a_n = (-1)^n \dfrac{n+2}{n^{-1/2}+3n}$

3. Express the number as a ratio of integers.

7.1363636....

4. Find the sum of the series

$$1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \frac{1}{25} - \dots$$

with an accuracy of 0.02.

5. Determine whether the given SERIES is convergent or divergent. Justify your answer! You *don't* need to find the sum.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{2n+3}{n+\sqrt{n}-1}$

(b) $\displaystyle\sum_{n=1}^{\infty} ne^{-n}$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \sin(1/n)$

(*d*) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{3^n}$

(*e*) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}-1}{n^2+n}$

6. Sketch the region bounded by the curves

$$y = x^2, \quad y = 0, \quad x = 0, \quad x = 2,$$

and compute the volume of the solid obtained by rotating this region about the x-axis.