

MA 125 CALCULUS I FALL 2006

December 08, 2006

FINAL EXAM

Name (Print last name first):

Student ID Number:

Instructor: Section:

PART I

Part I consists of 10 questions. Place your answer on the answer-line next to the question. Space is provided between questions for you to work each question (if you wish). No partial credit is awarded on Part I problems, only your entry on the answer line will be graded.

Each question is worth 4 points.

Question 1

Evaluate $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$. Answer:

Question 2

Evaluate $\lim_{x \rightarrow 0} \frac{e^x - x - 1}{x^2}$. Answer:

Question 3

For what numerical value of a is the function

$$f(x) = \begin{cases} 3x^2 + a - 1 & \text{if } x \leq 2 \\ ax & \text{if } x > 2 \end{cases}$$

continuous for all x ?

Answer:

Question 4

Find the value of x for which the curve $y = x - \ln x$ has a horizontal tangent?

Answer:

Question 5

Find an equation of the tangent line to the curve $y = x^2 - x$ at the point $(2, 2)$.

Answer:

Question 6

Let $h(x) = f(g(x))$, where $g'(2) = 4$, $g(2) = 3$, and $f'(3) = -2$. Find $h'(2)$.

Answer:

Question 7

Find all the critical numbers of the function $f(x) = 3x^{4/3} - 3x^{1/3}$.

Answer:

Question 8

Find the open interval(s) on which the function $f(x) = \frac{e^x}{x-1}$ is increasing.

Answer:

Question 9

Find all inflection points of the curve $y = x^4 - 6x^2$. [Be sure to give the x and the y coordinates of each point!]

Answer:

Question 10

Find the most general antiderivative of $f(x) = e^x - \sin x + \sec^2 x$ on the interval $(-\pi/2, \pi/2)$.

Answer:

PART II

Each problem is worth 10 points.

Part II consists of 6 problems. You must show the relevant work on this part of the test to get full credit; that is, your solution must include enough detail to justify any conclusions you reach in answering the question. Partial credit may be awarded on Part II problems where it is warranted.

Problem 1

Consider the equation

$$3x^2 + 2xy + y^2 = 9$$

in which y is implicitly defined as a function of x .

- (a) Use implicit differentiation to find y' .
- (b) Is the curve $3x^2 + 2xy + y^2 = 9$ rising or falling at the point $(2, -1)$? (Justify your answer!)
- (c) Find an equation of the tangent line to the curve $3x^2 + 2xy + y^2 = 9$ at the point $(2, -1)$.

Problem 2

Consider the function

$$f(x) = \frac{1}{4}x^3 - 3x + 1.$$

- (a) Find each open interval where $f(x)$ is increasing (you should find two intervals in all), and the open interval where it is decreasing.
- (b) Find all local maximum and minimum points of $f(x)$. [Be sure to give the x and the y coordinate of each point!]
- (c) Find the open interval where $f(x)$ is concave down, and the open interval where it is concave up.
- (d) Find the inflection point of $f(x)$. [Be sure to give the x and the y coordinate!]
- (e) Sketch a graph of $f(x) = \frac{1}{4}x^3 - 3x + 1$. (Clearly indicating the relevant items above on your graph.)

Problem 3

Consider the function

$$f(x) = \sqrt[4]{x}.$$

(a) Find the linearization (or linear approximation) of $f(x)$ at $a = 1$.

(b) Use the linearization of $f(x)$ to find an approximation of $\sqrt[4]{1.1}$.

(c) Another way to find an approximation of $\sqrt[4]{1.1}$ is to use Newton's method to find a root of the equation $x^4 - 1.1 = 0$.

Use Newton's method with initial approximation $x_1 = 1$ to find x_2 , the second approximation to the root of the equation

$$x^4 - 1.1 = 0.$$

Problem 5

A farmer wants to fence off a rectangular field that borders a straight river (with no fencing along the river). The field is to have an area of 200 m^2 . What are the dimensions of the field which use the least amount of fencing?

Problem 6

Use antiderivatives to answer the following questions.

- (a) Find $f(x)$ on $(0, \infty)$ if it is known that $f'(x) = \frac{x^2 - x + 1}{x}$ and that $f(1) = 0$.

- (b) Find the most general antiderivative $F(x)$ of the function

$$f(x) = \frac{1}{1 + x^2},$$

and then evaluate the expression

$$F(1) - F(0).$$

Summary of scores on problems - for grading purposes only. Do not enter any problem solutions or work on this page.

	Points
Part I - Question 1	
Question 2	
Question 3	
Question 4	
Question 5	
Question 6	
Question 7	
Question 8	
Question 9	
Question 10	
Part II - Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Problem 6	
Total Test Score	