MA 126-6A, CALCULUS II
December 03, 2007

Name (Print last name first): ....

Student ID Number (last four digits): ....

TEST IV

No calculators are permitted!

PART I - Basic Skills

Each question is worth 5 points.

Part I consists of 6 questions. Clearly write your answer (only) in the space provided after each question. You need not show your work for this part of the test. No partial credit is awarded for this part of the test!

Question 1

Determine whether the **sequence** $a_n = (-1)^n \dfrac{n+6}{5n-1}$ converges or diverges. If it converges, find its limit.

Answer: ....

Question 2

Find the limit of the **sequence** given by $a_n = \sin\left(\dfrac{3}{n}\right)$. (Your answer must be a number!)

Answer: ....

Question 3

Find the (numerical value of the) sum of the **geometric series** $\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^{n-1}$. (Your answer must be a number!)

Answer: ....

Question 4

Determine whether the infinite **series** $\sum_{n=1}^{\infty} \frac{n^2}{3n^2+45}$ is convergent or divergent.

Answer: ....

Question 5

Determine whether the infinite **series** $\sum_{n=1}^{\infty} \frac{1}{n^3}$ is convergent or divergent.

Answer: ....

Question 6

Determine whether the infinite **series** $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ is divergent, absolutely convergent, or conditionally convergent.

Answer: ....

PART II - Problem Solving skills

Each problem is worth 14 points.

Part II consists of 5 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit.

Problem 1

(a) Determine whether the **sequence**

$$a_n = \frac{5\ln n}{2n}$$

converges or diverges. If it converges, find its limit.

(b) Find the limit of the convergent **sequence** defined by

$$a_1 = 5, \qquad a_{n+1} = 2 - \frac{1}{a_n}.$$

Problem 2

Find the values of x for which the infinite **series**

$$\sum_{n=1}^{\infty} \frac{(x-2)^{n-1}}{3^{n-1}}$$

converges? **Write your answer in interval notation!**

Problem 3

(a) Find the numerical value of c for which

$$\sum_{n=1}^{\infty} \frac{1}{(2+c)^n} = 2.$$

(Hint: Note that the series starts from $n = 1$.)

(b) Determine whether the **series**

$$\sum_{n=3}^{\infty} \frac{3}{n(\ln n)^2}.$$

is convergent or divergent. (You must specify which test you use and show your work!)

Problem 4

(a) Consider the (infinite) **series**

$$\sum_{n=1}^{\infty} \frac{3n^5}{15n^6 - 2}.$$

Determine whether the series is convergent or divergent. (You must justify your answer!)

(b) Consider the (infinite) **series**

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n^4}}.$$

Determine whether the series is absolutely convergent, conditionally convergent, or divergent. (Only one of these three choices will be accepted as an answer, and you must justify your choice!)

Problem 5

Consider the (infinite) **series**

$$\sum_{n=1}^{\infty} \frac{(-2)^n}{\sqrt{n}}.$$

Answer all the following questions.

(a) Determine whether the ratio test is conclusive or inconclusive. (Justify your answer!)

(b) Determine whether the series is absolutely convergent, conditionally convergent, or divergent. (Only one of these three choices will be accepted as an answer, and you must justify your choice!)

SCRATCH PAPER

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