FALL 2007 — MA 227 — FINAL

Name: ______________________________

1. Part I

There are 10 problems in Part I, each worth 4 points. Place your answer on the line below the question. In Part I, there is no need to show your work, since only your answer on the answer line will be graded.

(1) Compute the cross product of the vectors $\langle 1, 0, 1\rangle$ and $\langle 2, 1, -3\rangle$.

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(2) Find the gradient of the function $f(x, y) = \sqrt{x^2 + y^2}$.

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(3) What is the direction of steepest ascent of the function $h(x, y) = xy$ at the point $P(2, 1)$?

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(4) Find a parametrization for a circle with radius 2 centered at the origin.

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(5) Find an equation of the plane with normal $\langle 2, 3, -1\rangle$ containing the point $P(0, 1, -2)$.

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(6) Find the linearization $L(x, y)$ of $f(x, y) = x^2y$ at the point $(1, 1)$.

(7) Compute the Jacobian of the transformation $x = u^2 - 2v$, $y = 2u + v^2$.

(8) Evaluate $\iint_D 2dA$ where D is the triangular region with vertices $(0, 0)$, $(0, 1)$ and $(2, 0)$.

(9) Find rectangular coordinates of the point with spherical coordinates $r = 3$, $\phi = \pi$, $\theta = \pi/4$.

(10) Compute $\operatorname{div} \mathbf{F}$ when $\mathbf{F}(x, y, z) = \langle yz, \cos(xz), -\sin(xy)\rangle$.

2. Part II

There are 6 problems in Part II, each worth 10 points. On Part II problems show all your work! Your work, as well as the answer, will be graded. Your solution must include enough detail to justify any conclusions you reach in answering the question.

(1) A particle's acceleration is given by $\mathbf{a}(t) = 9\sin t\,\mathbf{i} + 9\cos t\,\mathbf{j}$. At $t = 0$ its velocity is $\mathbf{v}(0) = 3\,\mathbf{i} + 3\,\mathbf{k}$ and its position is $\mathbf{r}(0) = -9\,\mathbf{j}$.
(a) Find the particle's velocity $\mathbf{v}$ at any time.
(b) Find the particle's position $\mathbf{r}$ at any time.

(2) Use two different methods to evaluate the line integral $\int_C (y+2x)dx + xdy$, where C is the line segment from $(0,0)$ to $(2,1)$.

(a) By direct integration.

(b) By the fundamental theorem of line integrals, after finding a potential function for $\langle y+2x, x\rangle$, that is a scalar function f such that $\nabla f(x,y) = \langle y+2x, x\rangle$.

(3) Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y, z) = 8y - 3z$ subject to the constraint $16x^2 + 4y^2 + z^2 = 25$.

(4) Evaluate the following integral by reversing the order of integration.

$$\int_0^4 \int_{y^{1/2}}^2 x \exp(x^4)\, dx\, dy.$$

(5) A surface S is given parametrically by $\mathbf{r}(u,v) = \langle u+2v, 2u+v, 1-u\rangle$ where $0 \leq u \leq 1$ and $0 \leq v \leq 2$. Evaluate the surface integral

$$\iint_S (x+y+z)\,dS.$$

(6) Gauss' Theorem (or the divergence theorem) states that

$$\iint_S \mathbf{F} \cdot \mathbf{dS} = \iiint_R \operatorname{div} \mathbf{F} \, dV$$

when R is a region bounded by the surface S. Use the theorem to compute the flux of $\mathbf{F}(x, y, z) = \langle 12z\mathrm{e}^y, y^3, 3x^2z \rangle$ across the cylinder $x^2 + y^2 = 1$ and its lids $z = -2$ and $z = 3$.